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Starch and Unannealed Glass under the Polariscope. *By* WALTER BAILY*.

[Plates I.–IV.]

THERE are many bodies, of which a spherical grain of starch and a circular plate of unannealed glass may be taken as specimens, having an optical structure symmetrical about an axis through the body. The object of this paper is to investigate the state of the light which emerges from such a body, when monochromatic light in any state of polarization is sent through the body in the direction of the axis.

In fig. 1 (Plate I.) let S S′ and T T′ be drawn through R perpendicular to one another, and let U U′ and V V′ bisect the angles between them. Suppose a quarter-undulation plate to be fixed parallel to the paper, with its axes parallel to S S′ and T T′—and the light to be passed perpendicularly to the paper through a Nicol's prism having its axis perpendicular to the paper and its plane of polarization inclined at an angle ρ to the line S S′, then through the quarter-undulation plate, and then through the body, which is also to be placed with its axis perpendicular to the paper.

Let the paper represent a section of the light after it has emerged from the body. Take any point P and draw round it an ellipse representing the polarization of the light at P.

* Read June 28, 1878.

The state of the light will be completely determined if we know the angle (α) which the axis major of this ellipse makes with R P, the angle (β) which a line joining the extremities of the axes of the ellipse makes with the axis major, and the direction in which the rotation takes place.

Let the angle S R P $=\phi$; produce R P to X and draw P Y $\perp$ P X; take any point Q on the ellipse, let P Q $=r$, and the angle Q P X $=\theta$.

The resolved part of the vibration at P along R P has been retarded in passing through the body differently to the resolved part perpendicular to R P. Let the resolved part along R P have been retarded by a quantity σ more than the mean amount, and the other part have been retarded by the same quantity less than the mean amount. The quantity σ is a function of the distance R P.

Let $\sin t$ represent the vibration in the æther after the light has passed through the Nicol. This is equivalent to

$$\cos\rho \sin t \quad \parallel \text{SS}'$$

and

$$\sin\rho \sin t \quad \parallel \text{TT}'.$$

After passing through the quarter-undulation plate the vibrations become

$$\cos\rho \sin(t+45°) \quad \parallel \text{SS}'$$

and

$$\sin\rho \sin(t-45°) \quad \parallel \text{TT}'.$$

These may be written (the coefficient $\frac{1}{\sqrt{2}}$ being omitted)

$$\cos\rho(\sin t+\cos t) \quad \parallel \text{SS}'$$

and

$$\sin\rho(\sin t-\cos t) \quad \parallel \text{TT}'.$$

It is easy to show that these vibrations represent motion in an ellipse whose axes are parallel to SS′ and TT′, and the extremities of whose axes are joined by a line making an angle ρ with the axis major.

These vibrations are equivalent to

$$\cos(\rho-\phi)\sin t+\cos(\rho+\phi)\cos t \quad \parallel \text{PX}$$

and

$$\sin(\rho-\phi)\sin t-\sin(\rho+\phi)\cos t \quad \parallel \text{PY}.$$

After passing through the body the displacement $\parallel$ PX is $r\cos\theta$, and that $\parallel$ PY is $r\sin\theta$. Hence

$$r\cos\theta = \cos(\rho-\phi)\sin(t+\sigma) + \cos(\rho+\phi)\cos(t+\sigma),$$

$$r\sin\theta = \sin(\rho-\phi)\sin(t-\sigma) - \sin(\rho+\phi)\cos(t-\sigma);$$

which may be written

$$r\cos\theta = a\sin t + b\cos t, \quad \ldots \quad (1)$$

$$r\sin\theta = a'\sin t + b'\cos t, \quad \ldots \quad (2)$$

where

$$\begin{aligned} a &= \cos(\rho-\phi)\cos\sigma - \cos(\rho+\phi)\sin\sigma, \\ b &= \cos(\rho-\phi)\sin\sigma + \cos(\rho+\phi)\cos\sigma, \\ a' &= \sin(\rho-\phi)\cos\sigma - \sin(\rho+\phi)\sin\sigma, \\ b' &= -\sin(\rho-\phi)\sin\sigma - \sin(\rho+\phi)\cos\sigma. \end{aligned}$$

Differentiating (1) and (2) with respect to t, we have

$$\cos\theta\frac{dr}{dt} - \sin\theta r\frac{d\theta}{dt} = a\cos t - b\sin t, \quad \ldots \quad (3)$$

$$\sin\theta\frac{dr}{dt} + \cos\theta r\frac{d\theta}{dt} = a'\cos t - b'\sin t. \quad \ldots \quad (4)$$

Multiplying (3) by $r\sin\theta$, and (4) by $r\cos\theta$, and subtracting, we have

$$\begin{aligned} r^2\frac{d\theta}{dt} &= (a\sin t + b\cos t)(a'\cos t - b'\sin t) \\ &\quad -(a'\sin t + b'\cos t)(a\cos t - b\sin t); \end{aligned}$$

whence

$$\begin{aligned} r^2\frac{d\theta}{dt} &= a'b - ab' \\ &= \sin 2\rho\cos 2\sigma - \cos 2\rho\sin 2\sigma\sin 2\phi. \quad . \quad (5) \end{aligned}$$

Again, in (1), (2), (3), (4), let t have a value which makes r a maximum or minimum; then θ becomes α, and $\frac{dr}{dt}$ vanishes.

Eliminating r from (1) and (2), we get

$$(a\sin\alpha - a'\cos\alpha)\sin t = -(b\sin\alpha - b'\cos\alpha)\cos t;$$

and eliminating $r\frac{d\theta}{dt}$ from (3) and (4), we get

$$(b\cos\alpha + b'\sin\alpha)\sin t = (a\cos\alpha + a'\sin\alpha)\cos t.$$

Hence

$$\frac{a\sin\alpha - a'\cos\alpha}{b\cos\alpha + b'\sin\alpha} + \frac{b\sin\alpha - b'\cos\alpha}{a\cos\alpha + a'\sin\alpha} = 0,$$

or

$$2(aa' + bb') - (a^2 + b^2 - a'^2 - b'^2)\tan 2\alpha = 0;$$

that is,

$$\sin 2\rho \sin 2\sigma + \cos 2\rho \cos 2\sigma \sin 2\phi + \cos 2\rho \cos 2\phi \tan 2\alpha = 0. \quad (6)$$

Now $r^2 \frac{d\theta}{dt}$ is proportional to the area of the ellipse, as the period of vibration is constant; and the axes of the ellipse are proportional to $\sin\beta$ and $\cos\beta$, since the intensity of the light, and consequently the sum of the squares of the axes is constant over the whole section Hence the area of the ellipse is also proportional to $\sin\beta \,.\, \cos\beta$ (that is, to $\sin 2\beta$). Putting $\rho = 45°$ and $\sigma = 0$ in (5), we get $r^2 \frac{d\theta}{dt} = 1$. But in this case the emerging light is circularly polarized, and consequently $\beta = 45°$, whence $\sin 2\beta$ also equals unity. Hence in every case

$$r^2 \frac{d\theta}{dt} = \sin 2\beta.$$

Hence

$$\sin 2\rho \cos 2\sigma - \cos 2\rho \sin 2\sigma \sin 2\phi - \sin 2\beta = 0. \quad . \quad (7)$$

The locus of points at which the major axis of the ellipse of polarization is inclined at a constant angle to the radius RP will be called an "isoclinal line," and will be denoted by the symbol $K(\alpha)$, where α is this angle. $K(\alpha)$ and $K(\alpha - 90°)$ are both included in the equation (6), as that equation does not distinguish between major and minor axes.

The locus of points at which the line joining the extremities of the axes makes a constant angle with the major axis will be called an "isomorphal line," and will be denoted by the symbol $M(\beta)$, where β is this angle. Equation (7) is the equation to $M(\beta)$.

The direction of rotation of the æther is positive or negative according as $\sin 2\beta$ is positive or negative—that is, according as β is positive or negative, since we need only give β values lying between $\pm 45°$.

In figs. 2, 3, 4, 5 these loci are represented—the isomorphals by continuous lines, and the isoclinals by dotted lines. The Arabic numerals indicate the values of α in degrees, and the Roman numerals those of β in degrees.

The value given to σ is RP + 90°, a function which has been assumed solely for convenience in drawing the figures. The figures are drawn between the limits $\sigma = 180°$ and $\sigma = 360°$.

An extension beyond these limits would give merely a repetition of the portion within them, since an addition of 180° to the value of σ makes no difference in the equations to the loci. Directions from the centres of these figures will be referred to by means of the lines in fig. 1.

The values of ρ are in fig. 2, 45°, in fig. 3, 30°, in fig. 4, 15°, and in fig. 5, zero. Accordingly the incident light is circularly polarized in fig. 2, elliptically in figs. 3 and 4 (the eccentricity of the ellipse being less in fig. 3 than in fig. 4), and plane-polarized in fig. 5.

The isomorphals and isoclinals drawn are those for which the values of α and β are 45°, 30°, 15°, and zero. The thick continuous lines are branches of M(0), which is the locus of plane-polarized light. The round spots are loci of circularly polarized light, or $M(\pm 45)$.

The symbol $K(\pm\alpha)$ will be used to include $K(\alpha)$, $K(\alpha-90°)$, $K(-\alpha)$, and $K(90°-\alpha)$; its equation is

$$(\sin 2\rho \sin 2\sigma + \cos 2\rho \cos 2\sigma \sin 2\phi)^2 = \cos^2 2\rho \cos^2 2\phi \tan^2 2\alpha. \quad (8)$$

And the symbol $M(\pm\beta)$ will be used to include $M(\beta)$ and $M(-\beta)$; its equation is

$$(\sin 2\rho \cos 2\sigma - \cos 2\rho \sin 2\sigma \sin 2\phi)^2 = \sin^2 2\beta. \quad . \quad (9)$$

Putting $90°-\phi$, or $-90°-\phi$, for ϕ in these equations makes no change in the equations. Hence $K(\pm\alpha)$ and $M(\pm\beta)$ are each symmetrical with respect to UU′ and VV′.

Adding (8) and (9), we get

$$\cos^2 2\phi \,.\, \cos^2 2\rho = \cos^2 2\alpha \,.\, \cos^2 2\beta, \quad . \quad . \quad . \quad (10)$$

from which equation it appears that the intersections of $K(\pm\alpha)$ with $M(\pm\beta)$ all lie on the two diameters defined by the equation; and since α and β are interchangeable in the equation, we see that $K(\pm\gamma)$ and $M(\pm\delta)$ intersect on the same two diameters as $K(\pm\delta)$ and $M(\pm\gamma)$.

Let

$$\sigma = \left(\frac{n}{2} + \frac{1}{4}\right)\pi + \vartheta;$$

then $K(\pm\alpha)$ becomes

$$(\sin 2\rho \cos 2\vartheta - \cos 2\rho \sin 2\vartheta \sin 2\phi)^2 = \cos^2 2\phi \cos^2 2\rho \tan^2 2\alpha,$$

and $M(\pm\beta)$ becomes

$$(\sin 2\rho \sin 2\vartheta + \cos 2\rho \cos 2\vartheta \sin 2\phi)^2 = \sin^2 2\beta.$$

From these equations it appears that, in changing the sign of ϕ, we shall change only the sign and not the magnitude of ϑ; so that if we have drawn part of $K(\pm\alpha)$ or $M(\pm\alpha)$ between the limits $\phi=0$ and $\phi=45°$, we can draw the corresponding part of the curve between the limits $\phi=0$ and $\phi=-45°$. If σ' be the value of σ corresponding to $-\phi$,

$$\sigma'=\left(\frac{n}{2}+\frac{1}{4}\right)\pi-\vartheta,$$

and therefore

$$\sigma+\sigma'=\left(n+\frac{1}{2}\right)\pi. \quad . \quad . \quad . \quad . \quad . \quad (11)$$

Putting $\alpha=0$ and $\beta=0$ in (8) and (9), we get for K(0),

$$\sin 2\rho \sin 2\sigma+\cos 2\rho \cos 2\sigma \sin 2\phi=0\,; \quad . \quad . \quad (12)$$

and for M(0),

$$\sin 2\rho \cos 2\sigma-\cos 2\rho \sin 2\sigma \sin 2\phi=0. \quad . \quad . \quad (13)$$

Let s be the value of σ where a radius ϕ intersects a branch of M(0), and let $s+\xi$ be the value of σ where the same radius intersects a branch of $K(\pm\alpha)$; then by (8) we have

$$\{(\sin 2\rho \sin 2s+\cos 2\rho \cos 2s \sin 2\phi)\cos 2\xi$$
$$+(\sin 2\rho \cos 2s-\cos 2\rho \sin 2s \sin 2\phi)\sin 2\xi)\}^2$$
$$=\cos^2 2\rho \cos^2 2\phi \tan^2 2\alpha. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

But by (13) the coefficient of $\sin 2\xi=0$, and ξ is determined by $\cos 2\xi$ only, from which it appears that ξ has pairs of values of equal magnitude and opposite sign. Hence, if we have drawn a part of $K(\pm\alpha)$ on one side of a branch of M(0), we can draw the corresponding part on the other side of M(0).

A similar property of $M(\pm\beta)$ with respect to K(0) may be proved in the same manner.

Where $K(\pm\alpha)$ intersects M(0), ξ vanishes and the diameter becomes a tangent. Its position is determined by putting $\beta=0$ in (10). This gives

$$\cos^2 2\phi \cos^2 2\rho=\cos^2 2\alpha. \quad . \quad . \quad . \quad . \quad . \quad (15)$$

The position of the diameter tangents to $M(\pm\beta)$ is determined by putting $\alpha=0$ in the same equation. We get

$$\cos^2 2\phi \cos^2 2\rho=\cos^2 2\beta. \quad . \quad . \quad . \quad . \quad . \quad (16)$$

Hence the same diameters are tangents to $K(\pm\gamma)$, $M(\pm\gamma)$ at the points where they intersect M(0) and K(0) respectively.

If we put $\sigma \pm 45°$ for σ in K(0) we obtain M(0), and conversely. See equations (12) and (13). . . . . . . (17)

If we put $\sigma \pm 90°$ for σ in $K(\pm\alpha)$ or $M(\pm\beta)$, the equation is unaltered; and by this means from one branch of one of these curves we can obtain all other branches of the curve. (18)

We will now consider the forms of the loci, dealing first with figs. 3 and 4, in which the incident light is elliptically polarized.

To obtain the isomorphals, we have the equation (7) for $M(\beta)$ and (13) for M(0).

Putting $\beta = \rho$, we obtain the following equation to $M(\rho)$, viz.

$$\sin\sigma(\sin 2\rho \sin\sigma + \cos 2\rho \cos\sigma \sin 2\phi) = 0\,; \quad . \quad (19)$$

and putting $\beta = -\rho$, we obtain the following equation to $M(-\rho)$, viz.

$$\cos\sigma\,(\sin 2\rho \cos\sigma - \cos 2\rho \sin\sigma \sin 2\phi) = 0. \quad . \quad (20)$$

$M(\rho)$ therefore consists of circles for which $\sin\sigma = 0$, and ovals intersecting those circles on the diameters SS′ and TT′; and $M(-\rho)$ consists of circles for which $\cos\sigma = 0$, and ovals intersecting those circles also on SS′ and TT′. The outer circles in the figures are circles of $M(\rho)$; and the middle circle is one of the circles of $M(-\rho)$.

When $\beta = \pm 45°$, the light becomes circularly polarized, and therefore the value of α becomes indefinite; consequently in equation (6) we must have $\cos 2\phi = 0$. Putting this value of $\cos 2\phi$ in (7), we get the following values of ϕ, σ, and β at points where the polarization is circular:—

$$\left.\begin{array}{lll} \phi. & \sigma. & \beta. \\ +45°, & n\pi + \ \ 45° + \rho, & -45° \\ +45°, & n\pi + 135° + \rho, & +45° \\ -45°, & n\pi + \ \ 45° - \rho, & +45° \\ -45°, & n\pi + 135° - \rho, & -45°. \end{array}\right\} \quad . \quad (21)$$

These points will be called the "circular points." All the isoclinals pass through all the circular points. The sign of β indicates the direction of rotation of the æther. Where the sign is positive, the direction of rotation has not been altered by the passage of the light through the body; where the sign is negative, the direction of rotation has been reversed.

To draw the isomorphals :—

Mark the circular points by (21). Draw the circles of $M(\rho)$ and $M(-\rho)$ by (19) and (20). Draw one branch of $M(0)$ from $\phi=0$ to $\phi=45°$ by (13).

Obtain a branch of $K(0)$ by (17).

Obtain an oval of $M(\rho)$ between the above limits by (14).

Draw the part of a branch of $M(\beta)$, for example, M(XV.) in fig. 3, or M(XXX.) in fig. 4, which lies on one side of $M(0)$, and complete on the other side by (14).

Draw the remaining branches of $M(\pm\beta)$ between the above limits by (18).

Draw the isomorphals between $\phi=0$ and $\phi=-45°$ by (11), and complete the figure by means of the symmetry about UU' and VV'.

Write against the isomorphals the values of β, taking care to make the sign of β the same as that at the nearest circular point.

To draw the isoclinals :—

Equations (6) and (8) are not in a form available for calculation; but by solving (8) as a quadratic in $\sin 2\phi$, we obtain the equation to $K(\pm\alpha)$ in the form

$$\sin 2\rho \cos 2\sigma \sin 2\sigma \cos^2 2\alpha + \cos 2\rho(1-\sin^2 2\sigma \cos^2 2\alpha)\sin 2\phi$$
$$= \pm \sin 2\alpha(\cos^2 2\rho - \sin^2 2\sigma \cos^2 2\alpha)^{\frac{1}{2}}, \quad . \quad (22)$$

from which, by putting successive values for σ, we can obtain corresponding values of ϕ.

By putting $\alpha=\rho$, we obtain the equation to $K(\pm\rho)$, viz.

$$\sin 2\rho \cos 2\sigma - \cos 2\rho \sin 2\sigma \sin 2\phi = \pm \sin 2\phi. \quad . \quad (23)$$

We have already obtained a branch of $K(0)$. Draw from (23) the part of a branch of $K(\rho)$ which lies on one side of $M(0)$ between the limits $\phi=0$ and $\phi=45°$, and from (22) obtain a similar part of a branch of $K(\pm\alpha)$—for example, K(15) in fig. 3 and K(30) in fig. 4.

Complete these branches by (14).

Complete the other branches between the same limits by (18).

Draw the figure between $\phi=0$ and $\phi=-45°$ by (11), and complete the figure by the symmetry about UU' and VV'.

Draw straight lines in the direction UU' and VV' to give

$K(\pm 45)$, for which the equation is

$$\sin 2\phi = 0. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

To number the isoclinals :—

Note in (6) that when $\sigma = n\pi$, $\tan 2\alpha = -\tan 2\phi$, or $\alpha = -\phi$; and when $\sigma = (n+\frac{1}{2})\pi$, $\tan 2\alpha = \tan 2\phi$, or $\alpha = +\phi$. Hence the intersections of the isoclinals with the circles of $K(\rho)$ graduate that circle in the negative direction, and their intersections with the circle of $K(-\rho)$ graduate that circle in the positive direction. Graduate these circles accordingly, taking care to deduct 180° from the graduation when it exceeds 90°, and 360° from it when it exceeds 270°, and the readings will give the values of α.

The deductions are made to keep the readings low, and for the sake of symmetry.

From the figures 3 and 4 it appears that $M(\pm\rho)$ divides the figure into regions of two kinds: one kind, which I will call the "segments," contains all the points at which the light is more circularly polarized than the incident light; and the other kind, which I will call the "rings," contains all the points at which the light is more plane-polarized.

In the segments the isomorphals are closed curves surrounding the circular points; and in the rings the isomorphals are closed curves surrounding the centre of the figure.

It also appears that $K(\pm\rho)$ divides the figure into regions of two kinds—one containing all the points at which both the axes of the ellipse are inclined to the radius by a greater angle than ρ, and the other containing all the points at which one of the axes is inclined at a less angle than ρ. Both kinds of regions are four-cornered, and have two opposite corners on circular points; but in the first kind both these circular points lie on the same radius, and in the second the circular points lie on different radii. The isoclinals in each region pass from one circular point to the other.

Comparing fig. 3 with fig. 4, we see that as ρ increases, the segments become smaller, and the isomorphals in the rings become more circular. When $\rho = 45°$, as in fig. 2, the segments vanish, and the isomorphals all become circular, the equation to $M(\beta)$ becoming

$$\cos 2\sigma - \sin 2\beta = 0.$$

The circular points are retained in the figure to show its relation to the other figures; but the whole circles through them are loci of circularly polarized light.

The equation to $K(\pm\alpha)$ gives $\tan 2\alpha = \infty$;

$$\therefore \ \alpha = \pm 45^\circ.$$

The equation to $K(\pm\rho)$ becomes

$$\cos 2\sigma = \pm \sin 2\phi.$$

This curve is retained in the figure to show the continuity with the other figures—although, as the inclination is everywhere 45° or -45°, the points on the curve have no special properties. For the same reason the straight lines $K(\pm 45)$ are retained.

Again, comparing figs. 3 and 4, we see that as ρ diminishes, the segments increase; and at last, in fig. 5, where ρ vanishes, the segments fill the whole space and the rings vanish. Each closed curve of $M(0)$ is forced into a broken line consisting of quadrants of circles joined by pieces of diameters; and as these closed curves now touch at their angles, they form together a complete system of circles and diameters whose equation is, putting $\rho = 0$ in (13),

$$\sin 2\sigma \sin 2\phi = 0.$$

The isoclinal $K(0)$ also forms a system of circles and diameters; their equation is, from (12),

$$\cos 2\sigma \sin 2\phi = 0.$$

All the other isomorphals form closed curves round the circular points; their equation is, from (7),

$$\sin 2\sigma \sin 2\phi + \sin 2\beta = 0.$$

And all the other isoclinals pass from one circular point to another on the same radius; their equation is, from (6),

$$\cos 2\sigma \tan 2\phi + \tan 2\alpha = 0.$$

The diameters of $K(0)$ and $M(0)$ coincide; this is indicated in fig. 5 by the thick continuous line having dots on one side of it.

The figure may be drawn in a similar manner to that described for figs. 3 and 4.

The locus of plane-polarized light may be investigated without reference to the condition of the rest of the light, by drawing M(0) by equation (13), and marking the direction of polarization at successive points on it by equation (15).

In figs. 6, 7, and 8 the points through which short straight lines are drawn are points at which the light is plane-polarized; and the short straight lines through them show the direction of polarization at the points. The dotted lines connecting these points are loci of plane-polarized light. The centres about which small circles are drawn are points at which the light is circularly polarized; and in fig. 6 the dotted lines connecting these circles are loci of circularly polarized light. The signs within the circles indicate the direction of rotation of the æther; see (21).

In fig. 6, $\rho=45°$; in fig. 7, $\rho=22° \ 30'$; in fig. 8, $\rho=0$.

If the light be passed through an analyzing Nicol with its plane of polarization inclined at an angle ρ' to SS′, we can obtain the intensity of the light at any point as follows:—

The vibration along the major axis is $\cos\beta\cos t$, and that along the minor axis is $\sin\beta\sin t$; so that the vibration in the direction of the plane of polarization of the analyzer is

$$\cos(\rho'-\alpha)\cos\beta\cos t-\sin(\rho'-\alpha)\sin\beta\sin t.$$

Hence, if I is the intensity of the light after passing the analyzer,

$$\mathrm{I}=\cos^2(\rho'-\alpha)\cos^2\beta+\sin^2(\rho'-\alpha)\sin^2\beta,$$

$$2\mathrm{I}=1+\cos 2(\rho'-\alpha)\cos 2\beta.$$

The appearance of the light after passing an analyzer might be calculated from this equation, but can be inferred more readily by an inspection of the figures, which show its state before passing.

We notice that two dark spots will be seen on each branch of M(0), one at each extremity of a diameter, at the points where the vibration is perpendicular to the plane of the analyzer. The spots on the successive branches of M(0) will be alternately on a certain diameter and on the diameter perpendicular to it.

When the incident light is circularly polarized, these spots will move round in circles with unaltered appearance and at a

uniform rate as the analyzer is turned uniformly. See figs. 2 and 6.

When the incident light is elliptically polarized, the spots will move round the curves $M(0)$; but the rate and appearance will vary (see figs. 3, 4, and 7). For on the circles of $M(\rho)$ the major axis of the ellipse preserves a constant direction in space, since $\phi+\alpha=0$; but on the circles of $M(-\rho)$ the major axis rotates uniformly in space with an angular velocity double that of the radius, since $\phi-\alpha=0$. Hence in those portions of $M(0)$ which are near the circles of $M(\rho)$, the change in the direction of the vibration will be slow; so that in this part the spot will be elongated, and will move more rapidly than the analyzer is rotated: but in the parts of $M(0)$ near the circles of $M(-\rho)$, the change in the direction of the vibration will be rapid; so that in these parts the spot will be shortened, and will move more slowly than the analyzer is rotated.

When the incident light is plane-polarized (see figs. 5 and 8), the slow-changing parts of $M(0)$ have combined to form the inner and outer circles of the figure and the diameters $S\,S'$ and $T\,T'$. Along these lines the direction of vibration has no change, but remains constantly the same as that of the incident light; but on the middle circle of the figure, and on corresponding circles, the direction of vibration rotates uniformly with a velocity double that of the radius. Hence on the latter circles there will be spots moving uniformly round with a velocity double that of the analyzer; but on the other parts of the figure there will be no spots. However, when the spots on the latter circles reach the diameters, then the former circles and the diameters will become black.

If the light is not monochromatic, these appearances will not be so distinctly seen, as the absence of one colour will not occur exactly in the same place as the absence of another, since the position of the isomorphal and isoclinal lines depends upon the value of σ, and this will differ for different colours. But the position of diameters which give plane-polarized light in figs. 5 and 8, is not dependent on the value of σ; and hence with any light this cross will always appear

uncoloured, being black when the upper and lower Nicols are crossed, and in full light when they are parallel.

If σ is constant, the isomorphals and isoclinals become straight lines from the centre, and the state of the polarization may be conveniently represented by taking a series of points in a circle round the centre, and drawing about each point the ellipse of polarization at that point. The ellipse will show the polarization along the radius on which it lies. This is done in figures 9 to 13, in which σ is about 15°. In fig. 9, $\rho=45°$; in fig. 10, ρ lies between 45° and σ; in fig. 11, $\rho=\sigma$; in fig. 12, ρ lies between σ and zero; and in fig. 13, ρ is zero.

Suppose now the light to be passed through an analyzer placed with its plane of polarization in the direction TT'. When $\rho=45°$, the quadrants about UU' will be dark, and those about VV' will be light. They will gradually shade into one another, there being no black or full light. As ρ diminishes, UU' becomes darker until, when $\rho=\sigma$, UU' is black (see fig. 11). As ρ further diminishes, the black bar opens out into a dark oblique cross, neither bar of which is black; and when ρ becomes zero, this cross becomes rectangular and black. As ρ passes on to $-\sigma$, the cross becomes oblique and not black, and closes up into a black bar along VV'; and when ρ becomes $-45°$, the quadrants about VV' are dark, and those about UU' light. When we have the oblique cross, we can by a suitable turn of the analyzer make either arm of the cross black. (See fig. 12.)

If the analyzer is placed with its plane of polarization in the direction SS', we get the same set of appearances, except that we get light for dark and dark for light; and in the case of the single bar and rectangular cross, we get full light instead of black.

The appearances presented when σ is variable may be well seen in cylindrical disks of unannealed glass. I do not know of any bodies which show very clearly the appearances presented when σ is constant. Crystals of salicene show the black cross remarkably well, and give indications of the single black bar; but in this substance σ, though constant along each radius, varies in passing from one radius to another, and this completely hides the phenomena of the oblique cross.

However, in grains of *tous-les-mois* starch, phenomena closely analogous to those above described as presented when σ is constant may be easily observed under a moderately high power—the only difference in the phenomena being that, in consequence of the grain of starch being generally an unsymmetrical body, the lines are distorted, the black cross, for instance, being neither rectangular nor rectilinear. See "The Optical Properties of Starch," Phil. Mag. for August 1876.

II. *An easy Method for Adjusting the Collimator of a Spectroscope. By* ARTHUR SCHUSTER, *Ph.D., F.R.A.S.*

THE ordinary method for adjusting the collimator of a spectroscope for parallel rays is only applicable to the mean rays of an achromatic combination. At the extreme ends of the spectrum a readjustment has to be made. If the ultra-violet rays are observed, and if the lenses are of quartz, the ordinary method cannot be used. The following method is so simple that I cannot help thinking it has often been in use ; yet I have nowhere seen it described, and I know that others, like myself, have often found a difficulty in making the adjustment without much loss of time and with simple apparatus.

The adjustment, as the following consideration will show, can be made on each line of the spectrum without any apparatus whatever. The only requirement is that the prism should be movable.

Suppose the rays which fall on the prism to be either convergent or divergent; then, after their passage through the prism they will seem either to converge to or diverge from a point, which is the secondary focus: as the prism is turned, so as to change the first angle of incidence, the secondary focus will change. If the rays are strictly parallel, then, whatever be the position of the prism, the focus will not be altered. This, then, is a delicate test for ascertaining whether rays proceeding from the collimator are parallel or not. It remains to be shown how it can be converted into a rapid method to put the collimator into the right adjustment.

The three fundamental equations for the passage of a ray of light through a prism,

$$\sin i = n \sin r, \quad \ldots \ldots \quad (1)$$

$$\sin i' = n \sin r', \quad \ldots \ldots \quad (2)$$

$$r + r' = \alpha, \quad \ldots \ldots \quad (3)$$

give

$$\frac{di'}{di} = -\frac{\cos i \cos r'}{\cos i' \cos r}. \quad \ldots \ldots \quad (4)$$

In these equations i and i' are the angles which the ray makes with the first and second surfaces respectively on entering and leaving the prism; r and r' the two corresponding angles of refraction, and α the angle of the prism. The right-hand side of equation (4) will, as a little reflection will show, steadily decrease when i is increased from 0 to $\frac{\pi}{2}$. This shows that the greater the first angle of incidence the more nearly parallel are the rays. The following system of consecutive approximation will therefore give the desired result.

Suppose the collimator is out of adjustment: move the telescope slightly out of position of minimum deviation; then two positions of the prism exist which will bring the desired ray into the middle of the field. Call the position in which the first angle of incidence is greatest A, the other B.

1. Put the prism into the position A, and focus the *telescope* until the line in question, either dark or bright, is distinctly seen.

2. Move the prism into position B, and focus the *collimator* until the same line is distinctly seen.

3. Repeat the operation, always focusing the telescope when the prism is in position A, and the collimator when the prism is in position B. After three or four trials no change of focus is required; both collimator and telescope will then be adjusted for parallel rays. I find that it is by no means necessary to work much out of the position of minimum deviation in order to gain a delicate adjustment. If the adjustment is made in the centre of the field, then I usually put the telescope into such a position that the line, when the prism is placed at maximum deviation, should just be out of the field

of view; this gives quite a sufficient change of focus if the rays are not parallel on entering the prism.

The following measurements, which were purposely made without special care, will show the accuracy of which the method is capable. The sliding tube of the collimator was divided into millimetres. Two different adjustments for the sodium-line, made in the way described above, gave the readings 5·0 and 4·0. The prism was now turned round so as to deflect the ray to the other side. Two adjustments now gave 4·1 and 5·0. The mean of the four readings is 4·5. The adjustment was then made according to the well-known method of first focusing the telescope on a distant object and focusing the collimator to the telescope afterwards: the reading was 4·2. As the focal length of the collimator was 300 millimetres, the two results differ only by a thousandth part of the focal length. Whether this difference is due to errors of observation, or whether it is produced by a difference in the focus of the yellow rays and the mean visible rays, I cannot say; but I believe, with a little precaution, the method can be adapted to the study of the achromatism of a lens.

I have assumed that the faces of the prism are perfectly plane. Practically it is difficult to get a prism in which this condition is accurately fulfilled; and it may be questioned whether the curvature of the prism may not seriously interfere with the accuracy of the method. To this I reply:—

1. That a prism which is known to be good may always be set aside to do this work.

2. That the reason of having the rays strictly parallel on entering the prism is based on the supposition that the faces of the prism are plane. It is by no means evident that parallel rays will give the best definition when the faces of the prism are curved.

3. That the change in the adjustment of the collimator introduced by the curvature of the prism is very small. One prism, which I know to be exceptionally bad, gave a difference of a half per cent. in the focal length of the collimator. It is not the change of focus introduced by the curvature of the prism which makes the method inaccurate when the prism is bad, but the difference in the change of focus in the two positions of the prism. This is one of the reasons why it is

better to take the two positions of the prism not too far away from minimum deviation. The small displacement of the prism will only introduce a small variation in the focal length due to the curvature of the faces.

III. *A Condenser of Variable Capacity, and a Total-Reflexion Experiment.* *By* C. V. Boys, *A.R.S.M., Lecturer for the Term on Natural Science at Uppingham School.*

Wishing to show my pupils the effect of condensation on the spark, I thought a condenser the capacity of which could be reduced gradually to nothing would be most suitable. So I made this simple contrivance, which answered its purpose well:—

A glass tube is sealed at one end and is covered with tinfoil for one third of its length; this forms the outer coating. The inner coating consists of a test-tube with the rim cut off, also covered with tinfoil; this is fixed to a wire, and can be drawn in and out. When it is fully in, the condenser has its maximum capacity; when drawn out as far as possible, the two coatings are too far apart to have any sensible action, and the capacity is zero.

On hanging this on the conductor of a Holtz machine the effect on the spark is well shown. Let the wire be first pushed in as far as possible, the condenser then acts to its full extent; but on gradually drawing it out the sparks are less and less bright, but follow one another more and more rapidly, till at last, when it is fully out, they have passed gradually to the almost continuous pale spark so characteristic of a Holtz machine. To show the effect best, the poles should not be more than about half an inch apart. Of course much ozone is formed inside the tube.

The total-reflexion experiment was an accident. A small condenser made of a test-tube gave way under the strain, a minute hole being pierced in the bottom, through which sparks passed almost continually. No light could be seen anywhere except on the rim of the tube, which formed a brilliant circle

of light. The light from the spark was totally internally reflected in the substance of the tube till it reached the rim, which it struck normally. The bright circle of light (the tube itself being dark) was very striking; and the experiment is a far truer illustration of total internal reflexion than the more beautiful one with a stream of water. The tube is, unfortunately, broken; and I have not succeeded in piercing another with the spark. A crack made with a hot wire does not do so well.

IV. *On the Music of Colour and Visible Motion.*

By Professors JOHN PERRY *and* W. E. AYRTON *.

[Plates V. & VI.]

AT the present time, when musical instruments of one form or another are employed nearly throughout the whole world, when even the emotions evoked by the sounds of the human voice have given life to the efforts of a whole nation in the 'Marseillaise,' we are apt to forget that our feelings may be excited through other media than sound. But, just as now all kinds of musical instruments are used in rendering the works of great composers, so we may expect that all known methods of exciting emotion will be combined in the grand emotional compositions of the future.

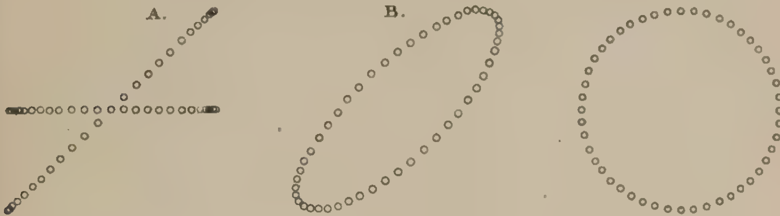
Although our feelings may be worked on through the medium of any of our senses, one only of these has been hitherto cultivated in the highest degree. And the reason of this is, that there exists an infinite number of *easy* ways of producing sound; so that combinations of sound have been used as the vehicle for exciting emotion in us, and in our forefathers, for the last four hundred years; and, as a result, the ear has been slowly trained to act as the conveyer of the varied impressions it is the province of the artist to create, whereas the means in our power of acting through the eyes are even up to the present day clumsy and inadequate.

Of the optical methods hitherto employed to work on the emotions, the oldest is certainly sculpture; but this can never

* Abstract of a paper read November 23rd, 1878.

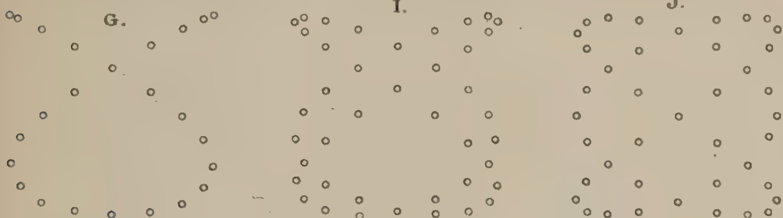
1 to 1.

A. B. C.



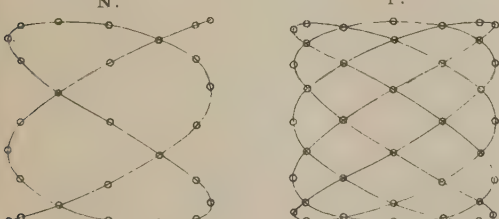
2 to 3.

G. I. J.



3 to 5.

N. P.

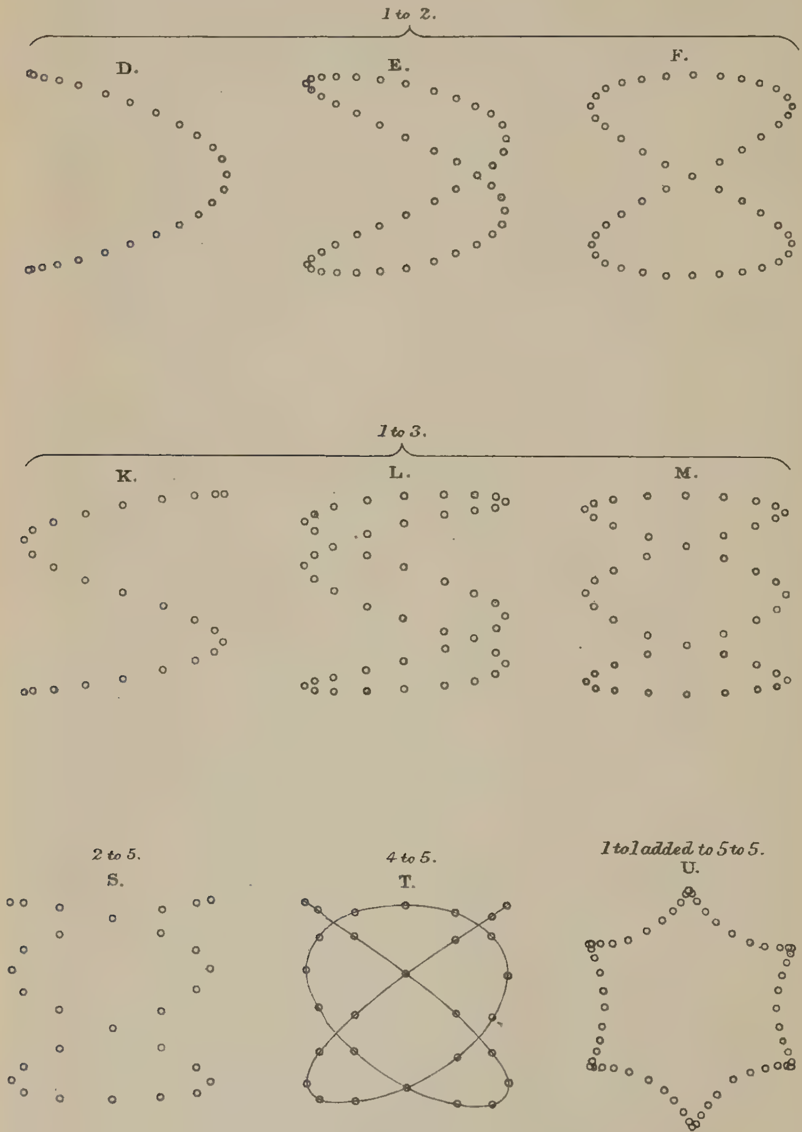


1 to 4.

Q.

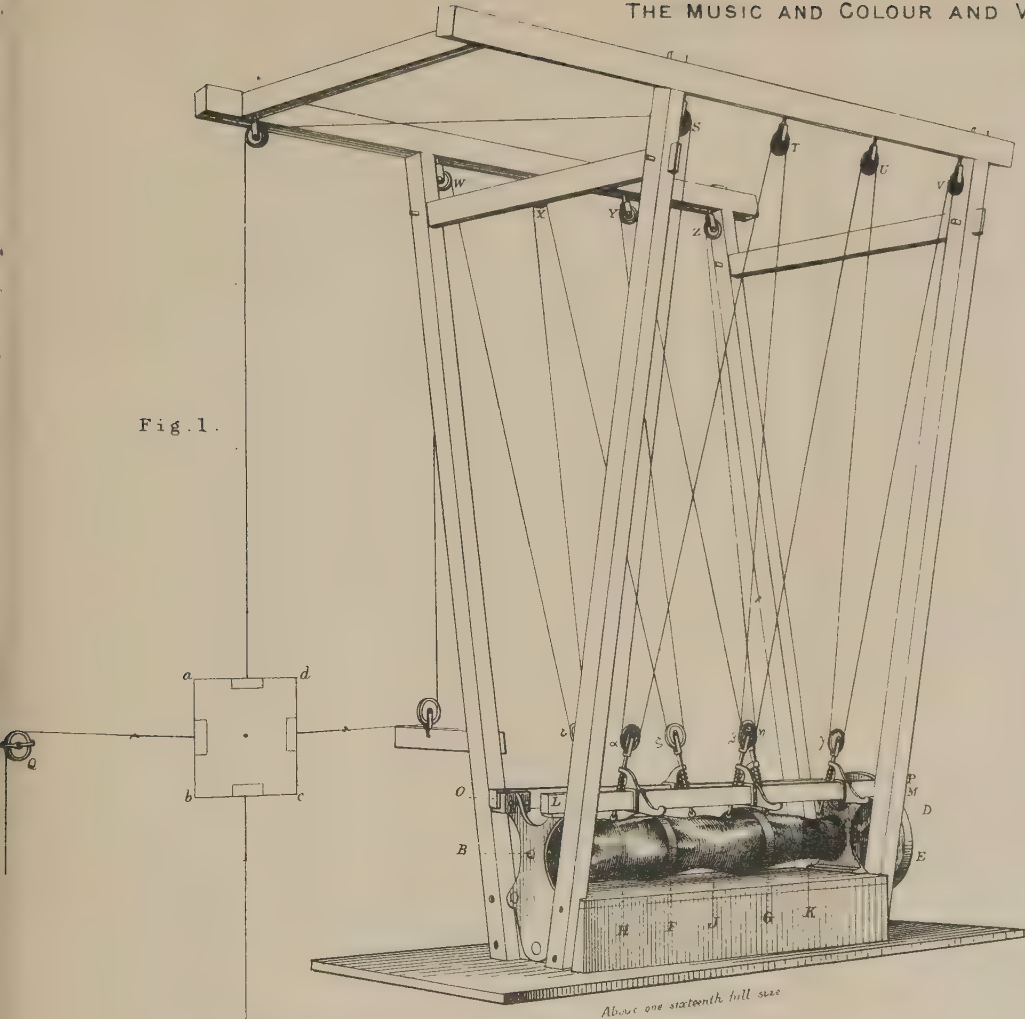
The distances between the small circles correspond

E MOTION. *(Professors Perry, and Ayrton.)*



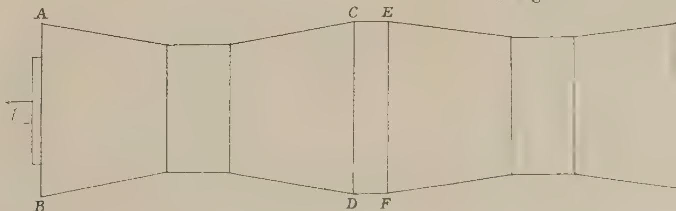
with equal times of description of the path.

Fig. 1.



About one sixteenth full size

Fig 5



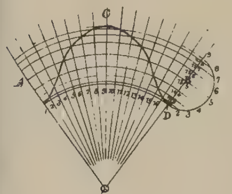


Fig. 2.

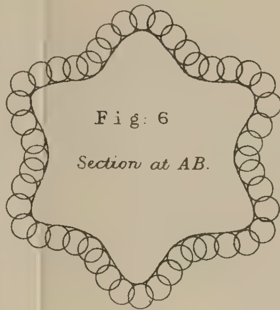


Fig: 6

Section at AB.

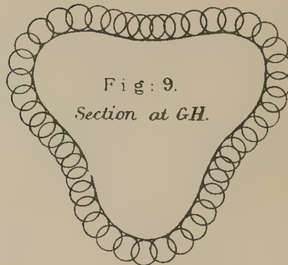


Fig: 9.

Section at GH.

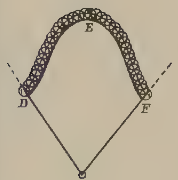


Fig: 3.

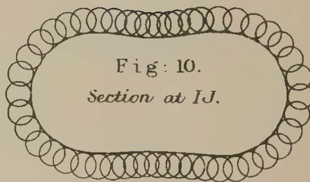


Fig: 10.

Section at IJ.

Fig: 7

Section at CD.

Fig: 11.

Section at KL.

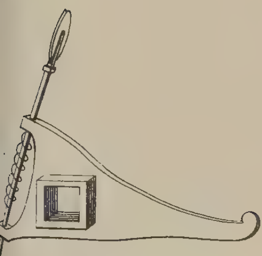


Fig: 4.

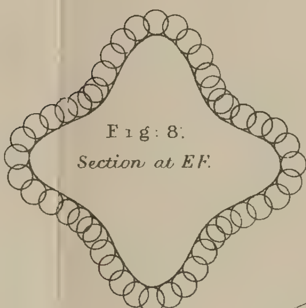


Fig: 8.

Section at EF.

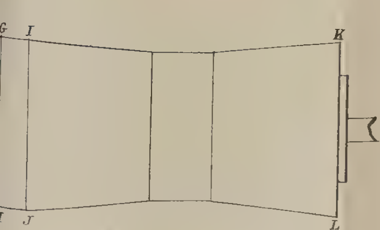
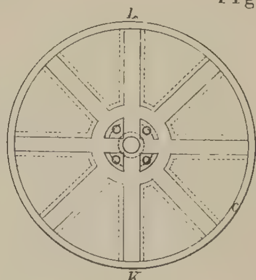
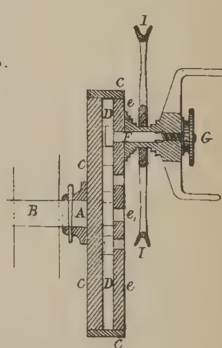


Fig: 13.





create an emotion unconnected with thought; and the feelings produced by it vary much in different people, and even in the same person at different times. The musical composer, on the other hand, is able to produce a definite succession of emotions which he can vary at will, and which are not utterly different in different people. A piece of sculpture is but suggestive—it merely introduces some simple emotion which acts in a controlling way upon the human mind; so also a fine picture induces a dreamy state and sets one a thinking.

Other emotions, again, are excited in particular people by certain associations connected with a taste, smell, &c.; but these may be likened to the energy stored up in gunpowder or nitroglycerine, to be set free by the smallest accident, whereas the efforts of real emotional art may be likened to those of which the effects may be calculated according to known laws.

For the eye to act as an agent for the emotions, it must receive much cultivation. Even with the experience of sound the ear has gained during the last four hundred years, how very few people are there sufficiently educated to have their feelings excited through music in answer to the emotions of a composer? and how pleasing to all is the *repetition* of a strain in a melody or the movement in a dance, perhaps from the instantaneous education the ear or the eye receives which enables it to better understand the movement when repeated. Consequently, if we consider the cumbrous and expensive nature of the apparatus necessary for producing regular changes of colour, or of motion, or of the size of the moving bodies, we may expect that it will probably take a long time before the world is able to employ our more complicated agencies.

It may appear at first sight that in placing motion on a footing of equality with what *we* consider its sister graces—sculpture, painting, and music,—we offered an indignity to these latter; and it may appear inconceivable to many how any amount of study of moving bodies can ever create an art as powerful and as enchanting as music. But it must be borne in mind that our present form of the fine arts probably only owes its existence to the accident that western nations have

more assiduously educated the emotional side of their minds in certain particular directions *.

And in our own country we have a close connexion between the varied emotions created by colour or movement and those excited by sound. It is well known that when certain persons hear an Oratorio, an Opera, or even a well-played violin solo, they see, without any voluntary effort on their part, beautiful changing mosaics, the patterns of which have definite connexions with the musical chords, and that such people always see a flash of light when they hear a sudden shriek of a railway-whistle.

The emotions excited by large bodies having a great velocity do not seem to be producible by any thing else in nature. These are felt when we stand on a bridge over a railway when a train approaches and passes underneath at great speed, or when we stand at the side of a railway when the train passes, even if we hear no sound, or if there is no appreciable trembling of the earth. In our first experience of such visible motion, terror, due perhaps to its utter strangeness, predominates. Similarly, the emotions produced by the play of colours in a fine sunset, by the rolling of the waves on the seashore, by the rhythmic motion of a large engine or of a long pendulum, by the tracing out of the combined harmonic curves, which have now become well known to all who have heard lectures on vibrating bodies, seem to be quite different from one another, and from the emotions produced by sound.

We have therefore tried to make a machine which to the emotions produced by a combination of colour, a mass in motion, and by motion in curved paths, would bear the same relationship as a musical instrument to music. We have not ventured to give our machine a name, since the name of the instrument should be that of the art; and although we have a name for exciting emotion by sound (music), we lack names both for the art of exciting emotion by colour, and by moving bodies.

* Here followed a number of examples taken from the Japanese stage and musical performances, proving that great conventionality existed among different nations in the expression of the emotions, and lending weight to the doctrine that music, unlike painting, received no suggestion from nature, and was therefore a creation of each individual people.

Many instruments have already been devised for combining together two harmonic motions; but as the conception of using such machines as emotion-exciters has never been present in the designers' minds, the performances of these instruments, although very beautiful, are necessarily of a comparatively elementary nature. The most important of these machines, all of which, with the exception of one or two Prof. Guthrie has been so very kind as to have arranged in working order before us, are Blackburn's pendulum, Wheatstone's kaleidophone, Lissajous' tuning-forks, Yeates's vibrating prisms, Donkin's harmonograph, Tisley's and Spiller's harmonograph, and Hopkins's electric diapason. In some of these, as in Blackburn's pendulum, only *one* particular pair of harmonic vibrations can be combined, and any change in the period of either means a stoppage of the instrument, corresponding in music with a delay in the tune at the end of every chord; in others we can change the period of one or other of the component harmonic vibrations, but have no certain means of controlling the amplitude, which in music would be equivalent to an inability to render at will a note *forte* or *piano*; or, rather, as it is not only the strength of the entire note, but even the amplitude of the various component harmonics that these instruments cannot regulate, it would be as if in music there was the probability of a note marked in the score as *piano* for the flute being rendered by a loud blast on the trumpet. In only one of these harmonic-motion compounders with which we are acquainted, viz. in the most perfect of the existing ones, Tisley's harmonograph, can an elliptic and a linear motion be combined; but even in this case a change from this motion to any other can only be made by first stopping the machine; and in none of the machines can a sudden change be produced in phase. Now it is obvious that we require a motion-producer of far greater range than any of these, if we are to play on the emotions, since all the qualities—elation and depression, velocity, intensity, variety, and form (which it is considered are possessed by a complex emotion)—must be visibly rendered.

The result produced by our instrument is this:—A round shadow is thrown upon the plain white wall of the room in which the audience are seated. The shadow appears to be a large black ball, of which the size during the performance may

be made to vary. It has motions over the wall which are pure harmonic or combinations of harmonic motion, produced by our being able to give to the shadow two independent motions —one in a vertical line, the other in a horizontal line, *each* consisting of a *combination* of linear harmonic motions, the *amplitude*, *period*, or *phase* of any one of which may be varied at will. We give a collection in figures (A to U, Plate V.) of some of the most simple paths traced out by our moving ball, and which differ from the ordinary Lissajous' figures in that we have placed little circles at such distances asunder as are passed over in *equal times*, in order to give an idea of how the velocity varies at different parts of any such path. Unfortunately, however, for giving a conception of the appearance, it is this variation of velocity, the effect of which on the senses no figure or description can give any idea, which constitutes one of the most striking features of the exhibition. Many of our readers will, however, have seen the motions of a very large Blackburn's pendulum, and of a very long simple pendulum; and they will therefore have gained such an idea of the motions of which we speak as a person, who has heard only the chirp of a sparrow, has about Beethoven's sonatas.

It must be remembered that while all the possible paths of the moving body are beautiful in shape, they are also endless in their variety. Not only may they vary without limit in their form, but also in their size, and in the velocity with which the body moves along them. Thus, perhaps the body is swinging slowly in a straight line in any direction, like the swaying of a huge tree in the wind, or so rapidly that a dark line only is visible, when, touching a key, the line is suddenly seen to open out, and the body rolls round a small or great circle, or a small or great ellipse of any proportions and with any velocity*; or the figure may be like any of the simple figures illustrated in the diagrams, or any others of the millions of different and more complicated forms producible. Some may change all their dimensions gradually in a certain direction, while dimensions in the other directions remain unchanged;

* One of the most beautiful things in connexion with the recent monster captive balloon in Paris was its rolling round and round in the breeze, like a huge inverted conical pendulum, after the stay ropes had been liberated, just before its ascent.

others may alter all their dimensions in different directions simultaneously, but with different velocities. Now, as we gaze at the body with all these graceful complex motions, which can readily be varied in shape or size of path, or in velocity of description, the same sort of awe comes over us as must have been felt by the people when they first listened to the strains of the earliest musical instrument.

To be sure it can never be as easy to change from a grave circular swing to a quick and complicated motion as it is to put down the key of a pianoforte; and it may be long before mechanicians will be able to let us vary the motion with sufficient rapidity. But we found that we could, even with our imperfect instrument, change one form of motion to almost any other possible one in about a second; and this is sufficiently quick for the present educational state of the art.

Our instrument itself we unfortunately cannot show, as it is the property of the Japanese Government, and is in Japan. Some photographs, however, of it and of our assistants who took part in its construction and performances are lying on the table. The exhibitions have been confined to ourselves and our students; but it was our intention, after we had educated ourselves by practising with the machine, to exhibit it publicly. Unfortunately, however, from various delays connected with improvements and alterations necessarily connected with the designing of such a new machine, it was hardly completed before one of the present writers left Japan, and the other has now no opportunities to practise with it. But it is our intention to construct a new instrument with many improvements on the first model, one of the most important being the carrying out of the idea we had at the commencement, of enabling the operator, by touching keys, to give any desired brilliant or sombre coloration to the wall on which the shadow of the body is moving, or to play on the wall a changing mosaic.

We will now describe the simplest form of our instrument, represented in fig. 1 (Plate VI.) about one sixteenth of its full size.

B C is a roller which is turned by a handle D (not visible in the figure), the fly-wheel E being of use in steadying the motion. The roller is divided into three portions, B F, F G,

G C, by the circular collars at F and G; at H, J, K, the centres of these portions, the section of the roller is circular, whereas at any other place it is such that, acting like a cam or tappet, it gives pure harmonic motion to a slider kept pressing on the roller, and only capable of radial motion. Such a sliding piece, resting any where between K and C, receives one complete harmonic motion during one revolution of the roller; if it rests anywhere between G and K, it completes two pure harmonic motions during a revolution, three between J and G, four between F and J, five between H and F, and six between B and H. The amplitude gradually decreases as the sliding piece is made to rest on places nearer the circular section, where of course there is no up and down motion of the sliding piece.

Every section, therefore, of the roller has a curved outline, of which the construction is easy. Thus suppose we want the section which will give a slider five complete harmonic swings in one revolution of the roller. Make the angle A O B (fig. 2) equal to one fifth of four right angles; describe the circular arc A B with O as centre and radius O A equal to $R+r$, where R is the radius of each of the three circular parts of the roller H, J, K, and r the radius of the small friction-wheel on the end of the sliding piece; make B D equal to half the maximum swing we wish the slider to receive. Then with centre B and radius B D describe the semicircle 159, which divide into any number of equal parts (eight in the figure), and let fall perpendiculars 22, 33, 44, &c. from the points of subdivision on to the diameter D B 9, meeting it in the numbered points. With O as centre, describe a circular arc through each of these numbered points in D B. Divide the angle A O B into twice the number of equal parts into which the semicircle 159 was divided. Then draw a curve 1 C D through the intersections of the first arc and first radius, the second arc and second radius, &c. Finally, draw a great many equal circles with radius r, the centres being in the curve; then the envelope D E F (fig. 3) is one fifth of the whole curved section we wish the curved roller to possess; and a template of tin may be made to be used in the construction of the roller. It will be found advisable to construct four templates for each division of the roller, since, although a section of the har-

monic surface described by the *centre* of the little friction-wheel formed by a plane passing through the axis of the large roller in a straight line, such a section of the actual surface on which the little wheel rests will be curved in consequence of r, its radius, not being infinitely small. Our large roller was made of hard wood; but it would have been much better if it had been made of cast iron or steel, since when of wood it, as well as the little friction-wheel of the sliders, must be made large to avoid abrasion; but even when these are large it is very difficult to avoid abrasion, and consequent slight irregularities in the motions of the slider, since the springs pressing down these sliders must be moderately strong to cause them to promptly follow all the alterations in curvature of the different parts of the roller.

Fig. 5 shows (reduced to a scale of one sixth) the roller as it came from the lathe before being cut to fit the templates; and figs. 6, 7, 8, 9, 10, 11 the sections at A B, C D, E F, G H, I J, K L (fig. 5) one quarter full size. Small circles have been drawn representing the little friction-wheel, and at such distances apart that in all cases the time taken for the wheel to pass from one position to the next is constant and equal to the forty-eighth part of the period of the revolution of the roller.

We used six sliders, α, β, γ, ϵ, ζ, η (fig. 1), one of which is shown enlarged in fig. 4. Each of these sliders could be moved longitudinally, parallel to the axis of the roller, along two stout iron bars L M, O P (fig. 1), in order to alter the amplitude of the swings. Sliders α and ϵ could be made to rest anywhere between B and F, β and ζ anywhere between F and G, and γ and η anywhere between G and C. Each slider carried at its upper end a large pulley made to move very easily (fig. 4); and the three pulleys of α, β, γ were always in the plane of the fixed pulleys T, U, V attached above to the wooden frame. A fine inextensible cord passing round the movable pulleys α, β, γ, and the fixed pulleys, S, T, U, V, was fixed at one extremity to the pulley V (the purpose of which was simply to adjust the length of the cord); and at the other end, where it hung vertically, it was attached to the top of the glass plate $a\,b\,c\,d$, from the bottom of which hung a weight in a pail of water to damp the motion of the

weight. This system then gave to the glass plate the sum of the motions of the sliders α, β, and γ. A similar cord over ϵ, ζ, η and W, X, Y, Z, together with the cord passing over the fixed pulley Q, and to which also hangs a weight in a pail of water, gives to the glass plate a horizontal motion equal to the sum of the harmonic motions of the sliders ϵ, ζ, η. It is evident, therefore, that to a circular patch stuck on the centre of the glass we were able to give motions compounded of the above harmonic motions perpendicular to one another, and by projection, by means of an electric light or heliostat, to cause this motion to appear like that of a large black ball rolling about on a white or coloured background.

Even still greater variety could have been imparted to the figures by the metal rods L M, O P (on which the sliders moved) having a motion at right angles to the radius of the roller. This could easily be arranged if the ends of the metal rods moved in circular grooves; and the result would be that not only could we alter the amplitude of any one of the component harmonic vibrations by moving a slider along the rod, but we could also alter the phase by giving the rod a circular motion. To do this, however, satisfactorily would have required either the employment of a much larger roller, so that the slowest vibration of a slider occurred four or five times during one revolution of the roller, or else a change in the arrangement. In the illustration (fig. 1) the glass plate, for simplicity is shown merely kept in position by the four cords; but in reality it moved in a horizontal frame which again slid in a vertical one, so that any lateral motion at right angles to the plane of the glass was impossible.

The reason why there is necessarily a considerable distance between the sets of fixed and moved pulleys is in order that a longitudinal motion of the slider shall not alter the *mean* horizontal or vertical position of the spot on the glass. At first we had the cords much longer than shown in the figure; but then, even after great care had been taken in endeavouring to obtain inextensible cords, some stretching was found to take place in practice; consequently we were compelled to determine experimentally what was the greatest length of cord that could be used without the stretching interfering with the accuracy of the motion; and this length was the one employed in the actual apparatus.

The ingenious way in which a number of pulleys are made to give the sum of their motions to the extremity of a cord was suggested to us by the arrangement employed in Sir W. Thomson's tide-calculating machine; but it is possible that in our new machine we shall adopt a totally different plan, and one which we think is new. If the two extremities of a long rigid rod have parallel motions perpendicular to the rod, the middle of the rod has a motion equal to half the sum of the extremities. Thus the parallel motions of two, four, or 2^n points may be compounded. Similarly for the three points, one third of the sum of parallel motions is obtained from the centre of a rigid triangular piece of which the points are the corners; so that by bars and frames of simple construction it is easy to get the sum of the parallel motions of any number of pieces.

We think the roller-plane has much to recommend it; but a series of little cranks may be better. Let a number of parallel shafts, having their bearings near their ends on two sides of a trough-shaped metal frame, receive from a system of spur-wheels near their centres relative velocities 1, 2, 3, &c. Let A (fig. 13) be either end of any shaft, B being the corresponding bearing; and let there be attached to it, and therefore revolving with it, a sort of double wheel of metal of the form shown. I I is the pulley round which the cord passes, if it is by means of a cord that we sum the harmonic motions; or the pulley may be absent; and the axle of the pulley is then simply a crank-pin. Then the rate of revolution of the shaft A determines the period, the distance of the axle F G from the shaft the amplitude, and the position of F G relative to a fixed diameter K L the epoch of the component harmonic motion. By shifting, then, the axle F G we may alter the epoch for any multiple of $\frac{\pi}{4}$ for the greater amplitudes, and of $\frac{\pi}{2}$ for the smaller. As there is, however, only a limited change of epoch, we think, on the whole, that our improved roller method is to be preferred.

We have used combinations of pure harmonic motions for obvious reasons; but it is possible that, instead of each section of the roller giving a pure harmonic motion, it may be found more suitable to have it giving some other kind of periodic motion; and such an instrument will differ from the preceding

in pretty much the same way that one musical instrument differs from another. As various means have already been devised, by revolving sheets of parti-coloured glass, for producing the effects of the chromotropes of the magic lantern, which physiologists have informed us produce such marked and instantaneous effects on the nervous constitution, and physical organs, of children, we have not yet specially turned our attention to the mechanical details of the colour portion of our machine.

In what has preceded we have spoken only of projecting the motion of a single ball on a wall; but there is no reason why the motions of several balls should not be gazed at simultaneously, nor why the people of a large city should not have an exhibition of the colour and motion art upon a canopy of clouds on a dark night.

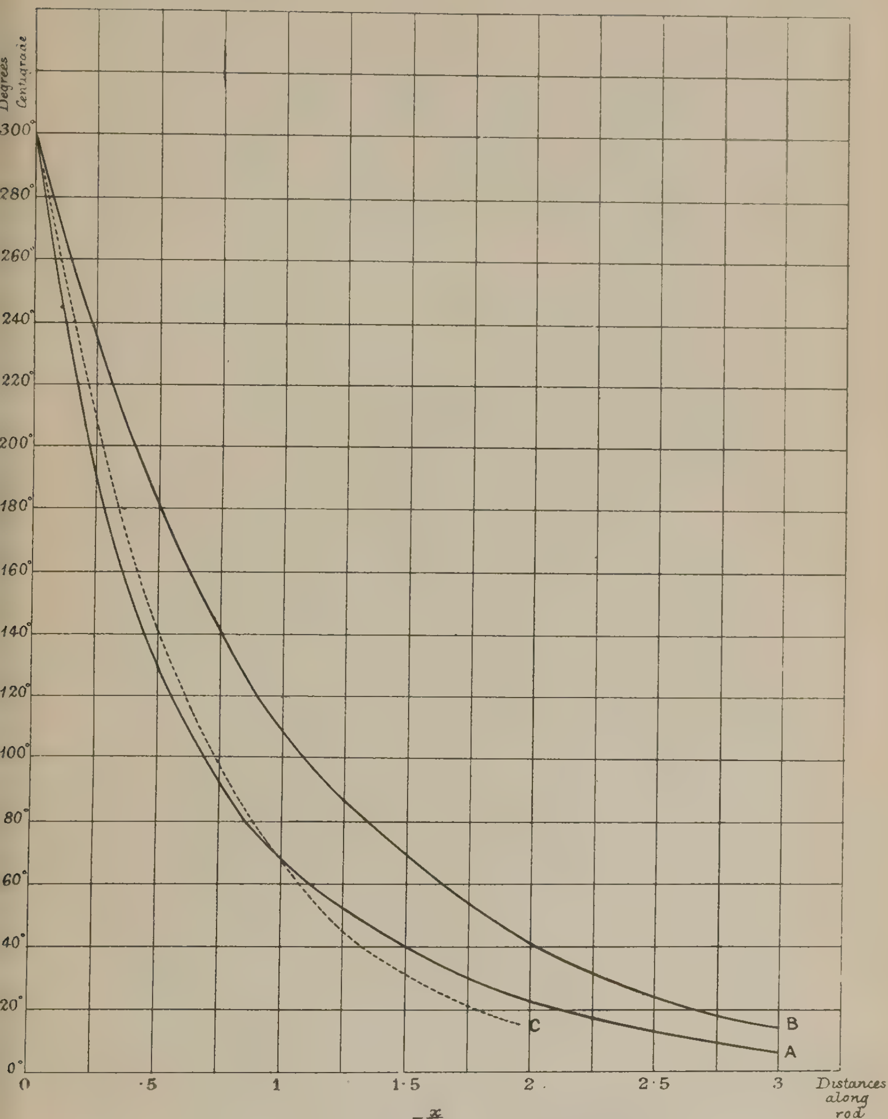
For assistance rendered us during the construction of this apparatus, and for the general intelligent interpretation of our wishes, we have to thank our late assistant, Mr. Kawaguchi, one of the brightest of the students of the Imperial College of Engineering, and one whose constant earnestness of purpose, while it rendered his life the more valuable to the scientific development of Japan, now makes his recent death the more to be deplored.

V. *On the Determination of the Variation of the Thermal Conductivity of Metals with Temperature, by means of the permanent Curve of Temperature along a uniform thin Rod heated at one end.* By OLIVER J. LODGE, *D.Sc., Lecturer on Applied Mathematics and Mechanics at Uuiversity College, London**.

[Plate VII.]

1. THE approximate theory of the flow of heat down a uniform rod heated at one end and exposed to cooling influences everywhere else was, I believe, given by Biot before the time of Fourier, and was also verified experimentally by him. The "constants" occurring in Biot's equation to the

* An abstract was read on the 8th of February, 1879.



Curve A is $y = \dfrac{300\ e^{-\frac{x}{80}}}{2 - e^{-\frac{x}{80}}}$ see equatim (20)

Curve B is $y = 300\ e^{-\frac{x}{80}}$

Curve C is $y = 300\ e^{-\frac{x}{53}}$ see § 21.

curve of temperature have long been known to be variable; and numerous experiments have been made to determine empirically in what manner they depend upon the temperature. Nevertheless Biot's method in its original form has been frequently employed by subsequent observers in order to compare the conductivities of different metals—by Ingenhousz for instance, by Despretz, and even in the far more accurate experiments of Professors Wiedemann and Franz. Principal Forbes, on the other hand, devised and executed a method which was quite independent of any equations to curves except those deduced from experiment; and by graphical and other laborious methods he determined both the absolute conductivity of wrought iron and its variation with temperature. But, as far as I know, no recalculation of the curve of temperature with the improved data now accessible has been made. It therefore seemed worth while to obtain as close an approximation to the equation of the true curve of temperature as is practicable without cumbrous integration, and to see how far the improvement affects the results of those experimenters who, unlike Forbes, depended on the theoretical curve of temperature. Moreover it seemed probable that the more accurate equation to the curve would enable the variation in conductivity of the rod with temperature to be calculated in some moderately simple manner, and with far less labour than that gone through by Forbes. Forbes's methods are perfect; the only objection to them is the excessive tediousness of the process of discussing the experimental results. And as it is a most important research at the present time to compare exactly the law of variation of thermal and electrical conductivity in the same piece of material, it seemed desirable to have some means of calculating the law of thermal variation from some simple experimental data; and the long-thin-rod form of experiment is evidently suitable for observing the variation of *electric* conductivity with temperature.

The following paper is unfortunately rather long; but the length is due to the necessity of fully discussing experimental results, and I have skipped nearly all the mathematical steps, as they are elementary and of no interest.

2. Consider a thin uniform infinitely long rod, of perimeter p and cross section q, made of a material whose specific con-

ductivity is k, density ρ, and specific heat c. Let this rod be surrounded by an enclosure at the absolute temperature v_0, and let one point of the rod (which we will call the origin) be kept by some means at a constant temperature Θ above that of the enclosure; then heat will flow from this point along the rod and will be dissipated at its surface, and the temperature of every point of the rod will rise at a rate proportional to the excess of the quantity of heat which it gains per second by conduction, over that which is dissipated by radiation and convection. After a long time, however, this excess of heat vanishes, and the temperature of any point of the rod ceases to rise, having attained a constant temperature θ above that of the enclosure—its absolute temperature, t, being therefore $\theta + v_0$. (I will adhere to the letter v for temperatures reckoned in Centigrade degrees from absolute zero, and to θ for temperatures reckoned from the temperature of the enclosure as zero. We shall have to use occasionally the Centigrade zero—the temperature of melting ice; and temperatures reckoned from it may be denoted by t.)

The heat which flows in unit time past any cross section of the rod at a distance x from the origin will be

$$-kq\frac{d\theta}{dx};$$

and the gain of heat per second by an element of volume qdx in this position will be the differential of this quantity, or

$$kq\,d\frac{d\theta}{dx}.$$

If every unit area of the surface of the rod at this point is losing by radiation and convection the quantity H per second, the rate of loss of heat by the surface of the element is

$$\mathrm{H}pdx,$$

the product pdx being the area of its surface. As long as the temperature of the element is rising, the rate of rise of temperature will be the difference of the last two expressions divided by the thermal capacity of the element—that is, divided by $cpqdx$; but when the permanent state is reached, the heat gained and the heat lost become equal, and their equality is the fundamental differential equation for the permanent state

of a rod, viz.

$$kqd\frac{d\theta}{dx}=\mathrm{H}pdx,$$

or

$$\frac{d^2\theta}{dx^2}=\frac{\mathrm{H}p}{kq}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (1)$$

The four quantities which enter into the right-hand side of this equation are all variables, and may be expressed as functions of θ. It has, however, been always assumed, in the approximate theory hitherto used, that H is the only variable, and that it is simply proportional to the excess of temperature, and can be written

$$\mathrm{H}=h\theta,$$

where h is a constant. (This is called Newton's law.) What we now want to do, however, is to take into account the variability of all these constants as far as present experimental results will enable us to do so, and then to integrate the above equation to as great a degree of accuracy as is easily possible.

3. Now, if an isolated body of volume V, surface S, density D, and specific heat C loses from each unit of surface a quantity of heat H per second, then its rate of fall of temperature is

$$-\frac{dv}{d\tau} \text{ or } \dot{v}=\frac{\mathrm{SH}}{\mathrm{VDC}};$$

writing $\dot{v}$ or $\dot{\theta}$ for the essentially positive quantity $-\frac{dv}{d\tau}$. Hence an element of the rod (§ 2), if isolated from its neighbours by two flat impervious films, will cool at the rate

$$\dot{\theta}=\frac{\mathrm{H}pdx}{c\rho\, dx},$$

whence its rate of loss of heat per unit of surface is

$$\mathrm{H}=\frac{qc\rho}{p}\dot{\theta}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (2)$$

Substituting this value of H in equation (1), it becomes

$$\frac{d^2\theta}{dx^2}=\frac{c\rho}{k}\dot{\theta}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (3)$$

which is precisely the *form* of the equation to the variable flow of heat through a slab*, though $\dot{\theta}$ has there a very diffe-

* See Everett, Trans. Roy. Soc. Edinb. vol. xxii.

rent meaning. The product $c\rho$ is the capacity for heat of unit volume of the rod (ρ being the mass of unit volume); hence $\frac{k}{c\rho}$ is the conductivity in terms of a unit of heat which can raise unit volume of the rod one degree. This Professor Maxwell calls the *thermometric* conductivity*, as distinguished from the calorimetric conductivity k.

4. In equation (3), $\dot{\theta}$ is a function of θ; and if the element were not supplied with heat, it would cool at the rate $\dot{\theta}$, and both θ and $\dot{\theta}$ would be functions of the time. But when heat is supplied to the element at a compensating rate by its neighbours, θ is constant, and therefore also $\dot{\theta}$ is constant as regards time; yet still the rod will emit heat at the same rate H as before, and $\dot{\theta}$ will be the same function of θ as if it were actually cooling: hence $\dot{\theta}$ was called by Forbes the *statical* rate of cooling.

The relation between $\dot{\theta}$ and θ for a cooling body, or the curve which expresses $\dot{\theta}$ as a function of θ, has been investigated experimentally by Dulong and Petit, and found to be of an exponential form. Newton's law made it a straight line. Forbes called it the secondary curve of cooling, and found a point of inflection on it for a long body cooling in air. For a rod in a permanent state, θ is a function of x; and the curve θ, x is the statical curve of temperature down the rod, and is the one we want to investigate. The curve $\dot{\theta}, x$ is what Forbes called the statical curve of cooling. Finally, the curve expressing θ as a function of time is the ordinary curve of cooling of a body. The general nature of these last three curves is the same, and depends on that of the first curve $\dot{\theta}, \theta$. The first rough approximation to them is that they are all logarithmic, this being a consequence of the hypothesis that the first curve is a straight line. I suppose that the fact that $\dot{\theta}$ is only *apparently* a function of the time renders abortive the analogy between equation (3) and the equation to the variable flow of heat in a slab.

On the Variation of $\frac{k}{c\rho}$ *with Temperature as at present known.*

5. Professor Tait has given theoretical reasons for assuming

* See Maxwell's 'Theory of Heat,' p. 235.

the conductivity (*i. e.* the *thermometric* conductivity) of every substance to be inversely proportional to the absolute temperature*; but I do not know whether he lays much stress upon the correctness of the theory. At any rate it does not seem to agree very well with the results of experiment, except in the case of iron. The experiments of Principal Forbes† established the fact that the thermometric conductivity of wrought iron is nearly inversely proportional to the absolute temperature; but the agreement is not quite perfect, as the following Table shows.

Centigrade temperature, t.	Conductivity at the temp. t, as found by Forbes, reduced to C.G.S. units, $\frac{k}{c\rho}$.	Product of conductivity and absolute temperature, $(274+t)\frac{k}{c\rho}$.	Product which is more nearly constant, $(400+t)\frac{k}{c\rho}$.	Product, $(308+t)\frac{k}{c\rho}$.
0°	·2331	63·87	93·24	72·26
50	·1995	64·65	89·78	71·82
100	·1764	65·98	88·20	72·32
150	·1629	69·08	89·60	74·93
200	·1528	72·41	91·68	
250	·1440	75·43	93·60	

The third column contains the numbers which ought to be constant if the theory were accurate. The fourth column contains numbers calculated on the hypothesis that the conductivity varies inversely as the absolute temperature increased by some constant, say by 126. These numbers agree with one another rather better than those in the preceding column; but still there is a regular divergence perceptible between the hypothesis and the experimental results, especially at high temperatures.

The results at the higher temperatures, however, do not seem to have been regarded by Principal Forbes as equally dependable; for he gives an empirical formula for the conductivity at any Centigrade temperature t which does not agree very closely with the experimental results at high temperatures, saying, "I have assumed that the most trustworthy

* See 'Recent Advances,' p. 271.

† Trans. Roy. Soc. Edinb. vols. xxiii. & xxiv.

part of the observational curves are those between the actual temperatures of 40° and 160°, and that within moderate limits the conductivity may be represented in terms of the temperature by such a formula as"

$$\frac{k}{c\rho}=A+Bt+Ct^2,$$

where the constants for the best of his two bars, when reduced to the C.G.S. system, are

$$A=\cdot 2331,\quad B=-\cdot 00755,\quad C=\cdot 00000189.$$

6. Forbes's formula may therefore be written

$$\left(\frac{k}{c\rho}\right)_{Fe}=\cdot 2331(1-\cdot 00324\,t+\cdot 0000081\,t^2);$$

and this may be very accurately expressed by a form more suitable for our present purpose, $\frac{A}{b+t}$. For this last may be regarded as the sum of an infinite geometrical progression with ratio $-\frac{t}{b}$, and may be written, without approximation,

$$\frac{A}{b+t}=\frac{A}{b}\left(1-\frac{t}{b}+\frac{t^2}{b^2}-\ldots+\frac{t^n}{(b+t)b^{n-1}}\right),$$

stopping at any term one likes, and multiplying it by $\frac{b}{b+t}$ instead of writing the remaining terms.

Now if T be the highest temperature to which the formula is required to apply, the average temperature $\frac{1}{2}$T may be introduced into the denominator of the last term instead of the variable t, without making much difference; and the above may be written approximately, stopping at the third term,

$$\frac{A}{b+t} \triangleq \frac{A}{b}\left(1-\frac{t}{b}+\frac{t^2}{b(b+\frac{1}{2}T)}\right)^*.$$

This expression agrees very well with Forbes's formula for a range of 200°; for taking $b=308$, $\frac{1}{2}T=100$, and $\frac{A}{b}=\cdot 2331$, it becomes

$$\frac{A}{308+t}=\cdot 2331(1-\cdot 00325\,t+\cdot 000008\,t^2).$$

Hence I shall assume that the results of Forbes may be

* The symbol $\triangleq$ is used merely to signify approximate equality.

summed up in the equation

$$\left(\frac{k}{c\rho}\right)_{\text{Fe}}=\frac{\text{A}}{308+t}=\frac{308}{308+t}\cdot\frac{k_0}{c_0\rho_0}. \quad . \quad . \quad . \quad (4')$$

The numbers in the last column of the preceding Table give the values of the "constant" A, for the temperatures considered most trustworthy by Forbes.

7. The variations of conductivity with temperature have also been investigated, by a method depending on fluctuating temperature, for copper and iron by Ångström*; and his result for iron, reduced to the C.G.S. scale, can be expressed in the following way,

$$\left(\frac{k}{c\rho}\right)_{\text{Fe}}=\cdot 2143(1-\cdot 002874\,t),$$

which may be considered as equivalent to $\frac{\text{A}'}{348+t}$ for a small range of temperature.

The results of Ångström for copper are summed up in the following formula,

$$\left(\frac{k}{c\rho}\right)_{\text{Cu}}=1\cdot 163(1-\cdot 001519\,t),$$

which may be expressed as $\frac{\text{B}}{658+t}$ when t is small.

8. The law of variation of the ordinary or calorimetric conductivity k can, of course, be deduced from the above by multiplying them by the value of the product $c\rho$, with each of its factors expressed as a function of the temperature—the one factor from the experiments of Bède on specific heat, the other from the expansion experiments of Fizeau. This Professor W. Dumas† has done. Using a mean coefficient of expansion between 0° and 100° C., he writes for the density of iron,

$$\rho=7\cdot 7799(1-\cdot 00003684\,t);$$

and for its specific heat,

$$c=\cdot 1053(1+\cdot 001348\,t).$$

Multiplying Ångström's value of $\frac{k}{c\rho}$ for iron by the product of

* Phil. Mag. vol. xxv. 1863.

† Pogg. *Ann.* cxxix. See also Wüllner, *Exp. Physik*, vol. iii. pp. 286 & 287.

these two quantities, we obtain

$$k = \cdot1862(1 - \cdot00156\,t).$$

And multiplying Forbes's numbers for $\frac{k}{c\rho}$ (nominally for k), as reduced to the C.G.S. system by Dr. Everett, on page 44 of his book on Units (first edition), by the variation factor $(1 + \cdot001311\,t)$, we obtain the real values of k at the different Centigrade temperatures t according to Forbes's experiments,

t.	k.
0	·207
25	·1975
50	·189
75	·182
100	·177
200	·171

numbers which can be expressed with considerable accuracy by the formula

$$k = \cdot207(1 - \cdot00144\,t).$$

9. The result of all this appears, then, to be that, as far as experiment has hitherto gone, the conductivity (both calorimetric and thermometric) of the metals copper and iron may be expressed with moderate correctness as a linear function of the Centigrade temperature, with a negative value for $\frac{dk}{dt}$, or

$$\frac{k}{c\rho} = \mathrm{A} - \mathrm{B}t,$$

and that it may be expressed with a trifle more accuracy by an inverse function of the temperature,

$$\frac{k}{c\rho} = \frac{\mathrm{A}}{b+t},$$

because this may be written very approximately, when b is much bigger than t,

$$\frac{\mathrm{A}}{b}\left(1 - \frac{t}{b} + \frac{t^2}{b^2}\right).$$

I will therefore assume that the variation of conductivity in any metal for moderate ranges of temperature is expressed by the equation

$$k = \frac{\mathrm{A}c\rho}{b+t}, \quad \dots\dots\dots \quad (4)$$

and that, since the variations of density and specific heat are known, the law of variation of conductivity in different metals is sufficiently discovered as soon as we have found the value of the constant b for each metal. Our object then is to find a mode of calculating b.

On the Variation of $\dot{\theta}$ with Temperature.

10. We have now to consider in what way the other factor of the right-hand side of equation (3), namely $\dot{\theta}$, may be expressed as a function of the temperature.

The magnificent researches of Dulong and Petit on this point have established the following expression for the velocity of cooling of a body whose absolute temperature is v, in an enclosure of absolute temperature v_0, containing gas at the pressure ϖ.

$$\dot{v}=\mathrm{P}(a^v-a^{v_0})+\mathrm{Q}\varpi^g(v-v_0)^{1\cdot232},$$

which may also be written in terms of the excess of temperature $\theta=v-v_0$, thus,

$$\dot{\theta}=\mathrm{P}a^{v_0}(a^\theta-1)+\mathrm{Q}\varpi^g\theta^{1\cdot232}. \quad . \quad . \quad . \quad . \quad (5)$$

The first term is the rate of cooling by *radiation*; the second term is the rate of cooling by *convection*. In other words, $\mathrm{Q}=0$ in a vacuum; and P is small if the surface of the body is silvered, but great if it be lampblacked. The constant g depends on the nature of the gas; for air it is ·45; but a is said to be a universal constant, and equal to 1·0077. Although this is an empirical formula, it is perhaps the most perfect example of such a formula that we have, and it expresses Dulong's results thoroughly well. The necessity of such an elaborate expression has been called in question by Narr; but his experiments, so far as they go, seem rather to confirm than to upset this expression; and it has been in the main verified by Provostaye and Desains.

11. Some caution, however, seems advisable with respect to the second term, which expresses the loss by convection as a constant power of the excess of temperature; because it was found by Principal Forbes that when the excess of temperature was very small, the loss by convection was almost inappreciable; and he suggested the viscosity of the air to account for this—some finite excess of temperature being required to

set convection-currents going. The point of inflection, moreover, which Forbes found on the curve $\dot{\theta}$, θ at a high temperature (see § 4) is wholly unaccounted for by Dulong's expression; but it is probable that here the experiments of Dulong are the most accurate, and that the contrary flexure of Forbes's curve was due to waves of heat in the elongated mass whose cooling he investigated*. Experiments on the rate of cooling of bodies at the ordinary pressure of the atmosphere have been made by Mr. Macfarlane and by Mr. Nichol; but their excesses of temperature only went as high as 60° (see Everett, 'On C.G.S. Units,' p. 50). I believe Professors Ayrton and Perry have some results not yet published.

On another ground also caution seems to be rendered necessary by the kinetic theory of gases, as illustrated in the experimental investigations of Mr. Crookes; for if the enclosure containing the cooling body be gradually exhausted of air, so that ϖ progressively diminishes, a discontinuity, in the direction of a sudden increase in the rate of cooling, would probably arise at the instant when the average free path of the molecules was long enough to reach from the surface of the cooling body to that of the enclosure. And it is probable that for exhaustions higher than this the law of cooling is different, and in all probability simpler than it was when the heat had to be conveyed between the surfaces by the unsystematic and irregular agency of convection-currents, a process of true gaseous conduction then setting in. This is a point which should be attended to in subsequent investigations; and it would be an important though somewhat difficult research to discover experimentally the law of cooling and its alteration with pressure when the distance between the cooling body and enclosure is less than the free path of the molecules: probably it could be more readily deduced from theory. It is not likely, however, that any of the investigators on the law of cooling hitherto have attained an exhaustion any thing like so perfect as this.

12. These objections, however, only apply to the *convection* part of the formula (5); and I will assume that the radiation part

$$\dot{\theta}=\mathrm{P}a^{v_0}(a^{\theta}-1) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5')$$

* Or see Professor Tait's explanation given to the Royal Society of Edinburgh on the 20th of last January ('Nature,' No. 486, p. 379).

is practically true as it stands. Since this, however, is not a very simple function for a second differential coefficient like (3), it will be well to see with what amount of accuracy we may expand it into a series and neglect higher terms. The expansion is

$$\dot{\theta}=\mathrm{P}a^{v_0}\left(\theta\log a+\frac{1}{2}(\theta\log a)^2+\frac{1}{6}(\theta\log a)^3+\frac{1}{24}(\theta\log a)^4+\dots\right) \quad (6)$$

which may be conveniently written

$$\dot{\theta}=\tfrac{1}{2}\mathrm{P}a^{v_0}(\log a)^2\cdot\theta\left(\frac{2}{\log a}+\theta+\frac{1}{3}\theta^2\log a+\frac{1}{12}\theta^3(\log a)^2+\dots\right),$$

or, putting in the numerical value of a, viz. 1·0077 (that is, putting $\log_e a=\cdot0076$),

$$\dot{\theta}=\mathrm{C}\theta(266\cdot\dot{6}+\theta+\cdot0025\,\theta^2+\cdot000005\,\theta^3+\dots),$$

or

$$\dot{\theta}=\mathrm{C}\theta\left(267+\theta+\frac{\theta^2}{400}+\frac{\theta^3}{200{,}000}+\frac{\theta^4}{18\times10^8}+\dots\right).$$

Remember that θ is to be ultimately the excess of the temperature of any point of the rod over that of the enclosure. It may be any thing between 0° and 150°; but it is not likely in ordinary experiments to go above 200°. The terms of the above series for the extreme case $\theta=200$ are

$$267+200+100+40+1,$$

where only the last term, containing the fourth power of the temperature, can be regarded as quite negligible. But for the more likely case of $\theta=100$, the terms of the series are

$$267+100+25+5+\frac{1}{18},$$

where the term containing the cube of θ is not of much consequence. If, however, it were wished not to go higher than the second power, the term containing the cube need not be neglected, but a mean value of it may be added to the coefficient of the second power of θ. Thus if Θ be the highest temperature taken notice of (*i. e.* the temperature of the origin in the case of the rod, the initial temperature in the case of a cooling body),

$$\dot{\theta}=\mathrm{C}\theta\left\{267+\theta+\theta^2\left(\frac{1}{400}+\frac{\frac{1}{2}\Theta}{200{,}000}\right)\right\};$$

and this is the expression we shall use, writing it first in the simpler form,

$$\dot{\theta}=\mathrm{C}\theta\left\{267+\theta+\frac{\theta^2}{400}\left(1+\frac{\Theta}{1000}\right)\right\}. \quad . \quad . \quad . \quad (7)$$

Notice that θ occurs in this expression as a factor, so that it is really a cubic function of θ.

It is singular how near the constant term in these brackets is to the number 274. I suppose this is accidental; but at first sight it looked as if the rate of cooling for small excesses of temperature were proportional to the product of absolute temperature and excess, or as if the quotient $\frac{\dot{\theta}}{v\theta}$ would be constant. On this hypothesis, however, the constant a, twice the reciprocal of whose logarithm is the number which happens to be nearly 274, would vary with v_0 the temperature of the enclosure, which is contrary to Dulong's results. Indeed there seems no ground for the conjecture.

Applying the correction for the neglected terms of the series, as is done in (7), we may write the expansion (6) thus, writing α instead of $\log_e a$ for shortness ($\alpha=\cdot0076$),

$$a^{\theta}-1 \doteq \alpha\theta\left\{1+\frac{1}{2}\alpha\theta+\frac{1}{6}\alpha^2\theta^2\left(1+\frac{1}{8}\alpha\Theta\right)\right\}. \quad . \quad . \quad (7')$$

13. It remains now to show, from Tables of experimental results, to what amount of accuracy θ, multiplied by a quadratic function of θ, will represent the observed rate of cooling of a body in a vacuum.

And first I will take the experiments of Narr* (see Wüllner, vol. iii. p. 254). The following Table contains the result of his experiments in a vacuum. The first column is the observed rate of cooling at the Centigrade temperature shown in the second column, the enclosure being at zero Centigrade. The third column contains the product of excess of temperature t and absolute temperature v, divided by the rate of cooling, to show how far this ratio is constant. These numbers are observed to decrease regularly, though slowly, and in a manner which has an obvious relation to the corresponding number in the preceding column; so that if twenty times that number be subtracted from each, the result will be very constant, as is shown in the last column.

* Pogg. *Ann.* vol. cxlii.

$\dot{t}$.	t.	$(274+t)\frac{t}{\dot{t}}$.	$\frac{vt}{\dot{t}}-20t$.
3·26	115°	13720	11420
3·11	110	13580	11380
2·80	100	13360	11360
2·49	90	13160	11360
2·18	80	12990	11390
1·88	70	12810	11410
1·73	65	12740	11440

Hence

$$\frac{vt}{\dot{t}}-20t=\text{const}=11400.$$

We may write this,

$$20\dot{t}=\frac{274+t}{570+t}\cdot t, \quad \ldots \ldots \quad (8')$$

or, approximately,

$$\dot{t} \doteq \frac{t}{11400}(274+t)\left(1-\frac{t}{570}\right),$$

which is of the form of equation (7), namely the excess of temperature t multiplied by a quadratic factor. The numerical value of the constants do not, indeed, agree well with those of Dulong, especially in the fact of the sign of the coefficient of t^2 being negative; but this is hardly to be expected, as Narr seems to have undertaken his experiments with the object of upsetting Dulong's results. Narr's experiments, moreover, do not extend over any thing like the range of temperature that Dulong and Petit's did.

14. If we apply the same process to the Table expressing the results of the latter experimenters in a vacuous enclosure at Centigrade zero, we shall find that the number $+30$ has to be used instead of -20; so that $\frac{vt}{\dot{t}}+30t$ is very tolerably constant, and equal to 18650 on the average, as is shown in the following abridged Table of Dulong's results. Considering that the range of temperature extends as high as 240°, the agreement is pretty good.

$\dot{\theta}$.	θ.	$\frac{(274+\theta)\theta}{\dot{\theta}}$.	$\frac{v\theta}{\dot{\theta}}+30\theta$.
10·69	240°	11530	18730
7·40	200	12810	18800
4·89	160	14200	19000
3·02	120	15650	18050
1·74	80	16270	18670

Hence we may write

$$30\dot{\theta}=\frac{274+\theta}{622-\theta}\cdot\theta, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8'')$$

or, approximately,

$$\dot{\theta}=\frac{\theta}{18650}(274+\theta)\left(1+\frac{\theta}{622}\right),$$

which is the form of equation (7).

It may be hereafter convenient to know that an expression like (8′) and (8″) is capable of representing the law of cooling in a vacuum with great accuracy, viz.

$$\dot{\theta}=\mathrm{C}\theta\cdot\frac{\mathrm{A}+\theta}{\mathrm{B}-\theta}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

but for our present purpose I think the equation (7) will be the most convenient.

15. The agreement of equation (7), as it stands, with Dulong and Petit's results it is scarcely necessary to show by a Table, since the equation has been deduced by known approximation from their own statement which completely expressed them, and the value of the terms neglected for an excess of temperature so high as 240° is perfectly evident. Nevertheless I have made the calculation, and the values of the "constant" $\left(267+\theta+\frac{6}{5}\cdot\frac{\theta^2}{400}\right)\frac{\theta}{\dot{\theta}}$, or $\frac{1}{\mathrm{C}}$, corresponding to the successive excesses of temperature 240°, 200°, 160°, 120°, and 80°, are 15265, 15865, 16489, 17086, and 16827. Hence the discrepancy between equation (7) and experiment is not *great* even for temperatures so high as 240°; while for a maximum temperature under 150° or so the discrepancy is practically *nil*.

[To be continued.]

VI. *On the Photographic Method of Registering Absorption-Spectra, and its Application to Solar Physics. By* Capt. W. de W. Abney, *R.E., F.R.S.*

There are certain difficulties in registering the visible absorption-spectra as observed, dependent on the eye of the observer, and on his power of representing correctly what he sees; and it is owing to these deficiencies that curious mistakes have been made in endeavouring to draw absorption-phenomena. Up to the present time it has been, comparatively speaking, useless to attempt such registration by means of photography, owing to the fact that merely one part of the spectrum was impressionable by the silver salts employed as a sensitive medium. Since my discovery that silver bromide could be prepared in such a molecular state as to be sensitive to the whole spectrum (visible, ultra violet, and ultra red*), the difficulty in the employment of photography is done away with; and it should be taken into use as much as possible, so as to eliminate the errors of eye-observations. A natural objection would arise at first sight, viz. that for the different parts of the spectrum the sensitiveness of the silver compound is materially different, and that consequently the absorption at different parts cannot be well compared. The objection vanishes, however, at once, if ordinary precautions are taken; and as an illustration I will take a case.

The absorption of a violet (cobalt) glass was required to be registered photographically. A spectroscope having two prisms of 62° was judged to give sufficient dispersion; and a lens was used in the camera of a focal length of about 2 feet. This gave a spectrum about 4 inches long, including the visible and invisible radiations. The plate having been placed in the camera, the top half of the slit was shielded, and sunlight was reflected onto the bottom half for two minutes; the sunlight was diverted, and the absorbing medium (in this case violet glass) was placed in front of the slit†, the lower half

* Except those radiations of low amplitude and large wave-length which are emitted by bodies at ordinary temperatures.

† When this paper was communicated to the Physical Society, Prof. Macleod suggested that the absorption of a liquid might be better demonstrated if a wedge-shaped vessel containing it were placed in front of a

covered up, and sunlight again reflected onto the top half of the slit for two minutes more. The plate was then developed, and a print taken from the negative. A scale of shade having been prepared, the following diagram was drawn from the measurements made with it.

The top continuous curve of fig. I. shows the intensity pro-

FIG. I

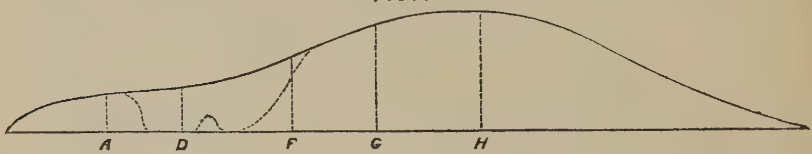
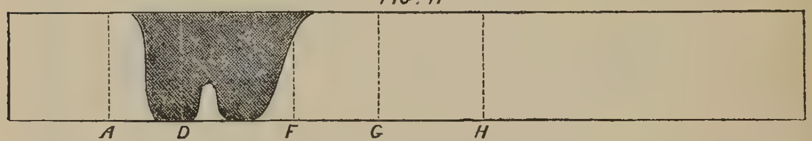


FIG. II



duced on the plate by unscreened sunlight. The dotted line in the same figure shows the curve obtained when the cobalt glass is interposed.

I would here remark that care is necessary not to introduce an error, as it must be remembered that the shades produced photographically have not the same gradations as the intensity of light, as Bunsen and Roscoe first showed.

Fig. II. shows the absorption of the violet glass, on the presumption that the intensity of the radiations is equal throughout the spectrum, an assumption which is very generally made.

I have found that it is convenient in taking these spectra to modify this method. The absorption produced by potassium chromate takes somewhat of a wedge-form, shading off from darkness in the violet to total transmission at the least-refrangible end of the spectrum. If a dilute solution of this

longer slit, of which a small image might be produced at the focus of the collimating lens. This is quite practicable, as Professor Macleod and myself have found by actual experiment; and if the image of coloured liquid be corrected by a similar wedge of colourless liquid of nearly the same specific gravity, there is no inconvenience attaching to it.

substance be interposed in each case between the source of light and the slit for half the time of exposure, we have an impression of the spectrum the varying intensity of which is less marked than if such an artifice be not employed.

I may here remark incidentally that the passage of light through an aqueous solution seems to interfere very little with the intensity of the photograph at the least-refrangible end. I had looked for a marked diminution, but have scarcely noticed it.

In photographing these absorption-spectra the source of light should be brilliant: sunlight, the image of the incandescent points of the electric light, or the oxyhydrogen light, may all be used; but I prefer sunlight, as we are enabled by the Fraunhofer lines to fix the locale of the absorption-bands more readily than with the other two.

Another application of this method is to the solar spectrum itself. Researches have shown that the bright-line spectra of incandescent compound bodies should lie in the least-refrangible end of the spectrum, and that to discover these a search must be made in these regions. As far as the visible spectrum is concerned such a search has been made; but we have yet to examine those regions which are invisible. At a low temperature it is quite possible that the compound bodies should give off vapours of the compound, whilst at high temperatures, such as that of the electric arc, they are probably dissociated. If, then, we wish to ascertain the existence of such compounds in the photosphere, we are driven to compare the solar spectrum with the bright-line spectra of the various compounds when heated at such low temperatures as those of the ordinary colourless gas- or spirit-flame. To photograph portions of such spectra (even the most "actinic" region of the spectrum) is a feat of uncommon difficulty; and it would require hours, I might say days, of exposure to impress lines in the red-region. Such an attempt would be practically useless, as we can accomplish the same end in as many minutes by an indirect method as it would require hours by the direct method.

The following illustration will show how it can be accomplished. The top half of the slit is covered as before, and sunlight reflected onto it, and the spectrum is impressed

on the photographic plate. The bottom half is next covered up, and a flame, in which the compound to be examined, is placed in front of the slit; the sunlight is then caused to traverse the flame, and a second spectrum is impressed on the plate through the top half of the slit.

New absorption-lines are thus formed in the solar spectrum, or *those already existent are intensified*, as is already well known. As an example, lithium chloride was heated in the flame, and the known line of lithium was found reversed between B and C, though absent in the spectrum of sunlight, and a faint line lying in the spectrum below the red was found intensified. By following out this plan we perhaps may eventually establish the existence of compounds in the solar photosphere. By using the light emanating from the white-hot carbon points of the magnetoelectric light to produce a continuous spectrum, and by burning the metallic compounds as before for one spectrum, and then by using sunlight to give the other spectrum, confirmatory evidence would be obtained. I may remark that I have photographed bright-line spectra of lithium, and got the same line in the ultra red as that obtained reversed. This method seems to promise to be a new weapon of attack in solar physics, more especially in this ultra-red portion.

VII. *On Spectra of Lightning.*

By Arthur Schuster, *Ph.D., F.R.A.S.**

All observers of lightning-spectra agree in having seen the line-spectrum of nitrogen; but most of them have seen, in addition to this, sometimes a continuous spectrum, sometimes a band spectrum, the chemical origin of which is unknown.

The following historical summary may give an idea of our knowledge on that point.

Prof. Kundt (Pogg. *Ann.* cxxxv. p. 315) observed a line spectrum consisting of one or two lines in the red, some very bright ones in the green, and some less bright ones in the blue. He mentions that the lines are not always seen together.

* Read February 22nd.

Lines which in one flash appeared especially bright were not seen in another flash. The greater number of flashes, however, gave a different spectrum altogether. In the place of bright lines a great number of bands were seen; and Prof. Kundt even distinguishes two different band spectra.

Mr. John Herschel (Proc. R. S. xvii. p. 61) observed a variable continuous spectrum crossed by bright lines, which also had a variable intensity. He gives the measurements of two lines, which agree very well with nitrogen-lines.

M. Laborde (*Les Mondes*, viii. p. 219) observed some lines, especially one near E, which sometimes appeared alone. He also saw a continuous spectrum.

Dr. H. Vogel (Pogg. *Ann.* cxliii. p. 653) describes lines only; but in his list I find two which do not coincide with any bright lines in the spectrum of the electric spark taken in atmospheric air; they do, however, coincide with two bands which I have observed in some flashes of lightning, as I shall show.

Mr. J. P. Joule ('Nature,' vol. xvi. p. 161) also observed some spectra of lightning. Frequently there was only one bright line visible, this being coincident with the brightest nitrogen-line. At other times there were several bright lines visible, sometimes with and sometimes without the green nitrogen-line. A continuous spectrum was also observed.

Mr. H. R. Procter ('Nature,' vol. xvi. pp. 161 & 220) gives some measurements of lines which do not lay claim to any accuracy. He observed also a band spectrum, which he finds not to be the band spectrum of nitrogen.

From conversation with Prof. A. Young, I learned that he also had seen a line spectrum, a band spectrum, and a continuous spectrum.

During my stay in Colorado last summer, I had some good opportunities of studying the spectra of lightning. It was my intention to get some reliable measurements of the band spectrum which I, in common with most observers, have seen; and in order to have greater chance of succeeding, I confined myself to one part of the spectrum only. The part I chose extended from $\lambda=5000$ to $\lambda=5800$, and covered, therefore, most of the yellow and green. I used a direct-vision spectroscope, with a slit movable by means of a micrometer-screw. A bright line in the principal focus of the telescope

formed a fiducial mark. Under ordinary circumstances, the slit is moved until the line to be measured forms a continuation of the bright line which reaches down into the centre of the field. I found, however, that the bands I wanted to measure were nearly as broad as the thin glass bar which carries the bright line; and I used the bar therefore simply as pointer. The measurements were always made at night; and the spectroscope was left undisturbed until the following morning, when the Fraunhofer lines in the neighbourhood were measured, so that the wave-lengths of the measurements could be interpolated.

It is of course impossible to put a pointer on a band during the instantaneous flash; but a succession of flashes allows us to put the pointer successively nearer and nearer until we see it in coincidence with the band. In this way several readings of each band were obtained. The dispersive power of the spectroscopes was such that, with a higher-power eyepiece than the one used in this investigation, the nickel-line could be seen between the two sodium-lines. The distance between the two sodium-lines was such that the two readings of the slit differed by ten divisions of the micrometer, or one tenth of a whole revolution. With sunlight I can measure easily to the tenth part of the distance between the sodium-lines. I obtained measurements on three different nights. Unfortunately, the best nights for the work occurred before the Total Solar Eclipse, which had taken me out to Colorado. A desire to save my eyes prevented me from making as good use of these nights as I otherwise should have done.

July 25th, West Las Animas.—The whole horizon seemed to be almost constantly illuminated with lightning, generally sheet-lightning. I observed about thirty or forty different flashes. I often saw the bright nitrogen-lines 5002 and 5681. I did not take any measurements of these lines; but there can hardly be a doubt as to their position. I saw in the part of the spectrum which I was observing three bands, which, however, did not always appear together. The measurements reduced to wave-lengths will be given further on. Two measurements of the bands β and γ were obtained, but one only of the band α. The greatest difference between the two measurements amounts to three times the distance between the sodium-lines. This

difference must be partly accounted for by the difficulty of the observation, partly by the fact that the spectroscope had only just been unpacked after the journey; and it was found next day that it was considerably out of adjustment. The micrometer-screw, also, owing to the heat and dust, had a considerable backlash; it was taken to pieces next day and cleaned, which greatly improved it.

August 3rd, Manitou.—Clouds were coming from the west over Pike's Peak; and strong flashes of lightning, partly sheet lightning, partly forked lightning, were observed. Only two measurements were secured. One of the bands measured was β. Prof. Arthur Wright, who was present, observed that one spot of the sky was illuminated during some flashes with a strong blue light, looking like a fluorescent light. I pointed the spectroscope to that spot, and observed a single broad band in the green. I moved the pointer on it as well as I could; but not being able to get another flash to verify the measurement, I had to take the reading. The position of this band, which I call δ, is very doubtful.

August 18th, Salt-Lake City.—I only obtained one measurement of the band γ. The kind of lightning observed differed considerably from that of the preceding nights. The lightning was nearly all forked lightning; and the bright nitrogen-line came out very strongly. The bands were but seldom seen. In one flash I saw a series of lines in the green which I had never seen before. My impression is that they were at about equal distances from each other, decreasing in strength towards the red; so that the whole made an impression similar to that of a fluted band, such as those seen in the spectrum of aluminium oxide, but shading off towards the red.

In addition to the line and band spectra, I have on many occasions seen a continuous spectrum only.

The following Table contains all the measurements I have taken. I have added in the last column numbers contained in Dr. Vogel's list of lines. It will be seen that these coincide with two of the bands I have seen.

Band.	Date.	λ	Mean.	Vogel.
α.	July 25	5592	5592	
β.	July 25 July 25 Aug. 3	5348 5329 5325	5334	5341
γ.	July 25 July 25 Aug. 18	5175 5193 5177	5182	5184
δ.	Aug. 3	5260	5260	

In trying to identify these bands with known spectra we meet with an unexpected difficulty. Two of them unfortunately admit of two different interpretations. At first sight I was struck by the close agreement of α and γ with two bands of carbonic oxide. These bands fade away towards the blue; and their sharp edges have a wave-length of 5607 and 5197. Observing with the same spectroscope, and widening the slit as I did in observing the lightning, I can produce the same impression of an unshaded band; and taking a measurement of the centres of the bands under these circumstances, I obtain $\lambda=5579$ and $\lambda=5180$, which agree within the limits of possible errors with the above values.

The ordinary spectrum of air, however, contains a band at 5178; so that, as far as mere position is concerned, one might well be taken for the other. I was, however, under the impression that I had sometimes seen this band without the chief nitrogen double line 5002–5; and as the yellow band of carbonic oxide was also apparently present, I stated with considerable confidence when I first wrote out this paper that I had observed the spectrum of carbonic oxide. It was only when I came to work out the position of the band δ that I began to have serious doubts as to the accuracy of this conclusion. The position of the band δ, as I have said, is very doubtful; I even thought it was possible that I had taken a very bad measurement of either β or γ, and felt at first inclined to reject it altogether. On working out its wave-length, however, I found that it was coincident with one of two strong bands, which are found at the negative pole of vacuum-tubes filled with oxygen. Now the second of the two bands is nearly coincident with the yellow band of carbonic oxide; so that, of the two bands

which I at first thought belonged to that gas, one might be due to nitrogen, the other to oxygen, as seen at the negative pole.

The explanation of the band β is obvious. It is the brightest of the two green lines in the low-temperature spectrum of oxygen. Its wave-length, when seen under a pressure of about a millimetre, is 5329; but under higher pressures it widens more on the less-refrangible side than towards the blue, and may well appear as a band with its centre at 5334 or even 5341, as given by Vogel.

I have not been able to obtain this band from atmospheric air in vacuum-tubes, although I have tried the experiment under various pressures. If the so-called continuous discharge is allowed to pass, the band spectrum of nitrogen alone appears; if the disruptive discharge passes, the high-temperature spectrum of oxygen is superadded to the line spectrum of nitrogen. As regards the two bands α and γ, it does not seem to me to be possible at present to decide between the two interpretations which I have given. On the one hand, it seems improbable that the slight traces of carbonic acid known to exist in the atmosphere should reveal their presence in the spectrum; but, on the other hand, it is to be remarked that oxygen vacuum-tubes, which show the band β, always reveal the slightest trace of carbonic oxide. It is exceedingly difficult, though quite possible, to obtain the band β without the bands α and γ. The measurement of δ, however uncertain, renders it probable that the spectrum of the negative pole in oxygen forms part of the spectrum of lightning; and on the whole I should feel inclined to attribute the band α to oxygen. I have shown in my paper on the spectrum of oxygen that this spectrum of the negative pole is due to an allotropic modification of oxygen (possibly ozone), and I have been able to obtain it, though only temporarily, in the positive part of the discharge. As regards the band γ, I have some difficulty in attributing it to nitrogen, and still think it probably due to carbonic oxide. During the observations I certainly felt convinced that it did not belong to the same spectrum as the chief lines of nitrogen, and I made a note that, on the contrary, it generally appeared together with β. It seemed sometimes to be present alone, and often to form the most prominent part of the whole spectrum. As the lines of the capillary part of an oxygen-tube are also present at the negative pole, together

with the bands distinctive of that pole, I can best express my observations on the band spectrum of lightning by saying that it resembles in a remarkable way the spectrum which is found at the negative pole of a vacuum-tube filled with oxygen which is slightly contaminated with carbonic oxide.

VIII. *On the Transmission and Distribution of Energy by the Electric Current.* *By* C. WILLIAM SIEMENS, *D.C.L., F.R.S.**

[Plate VIII.]

IN the autumn of 1876, when standing below the Falls of Niagara, the first impression of wonderment at the imposing spectacle before my eyes was followed by a desire to appreciate the amount of force thus eternally spent without producing any other result than to raise the temperature of the St. Lawrence a fraction of a degree†, by the concussion of the water against the rocks upon which it falls.

The rapids below the fall present a favourable opportunity of gauging the sectional area and the velocity of the river; and from these data I calculated that the fall represents energy equivalent to nearly 17 million horse-power, to produce which by steam would require about 260 million tons of coal a year, or just about the entire amount of coal raised throughout the world.

If one fall represents such a loss of power, what must be the aggregate loss throughout the world from similar causes? and is it consistent with utilitarian principles that such stores of energy should go almost entirely to waste? But the difficulty arises, how such energy (occurring as it does for the most part in mountainous countries) is to be conducted to centres of industry and population.

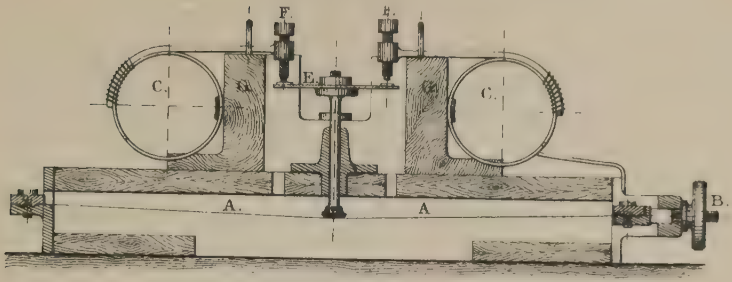
Transmission by hydraulic arrangements or by compressed air would be very costly and wasteful for great distances; but it occurred to me that large amounts of energy, produced by means of the dynamo-electric current-generator, might be conveyed through a metallic conductor, such as a rod of copper fixed upon insulating supports. Such a conductor would no doubt be expensive; but, if once established, the

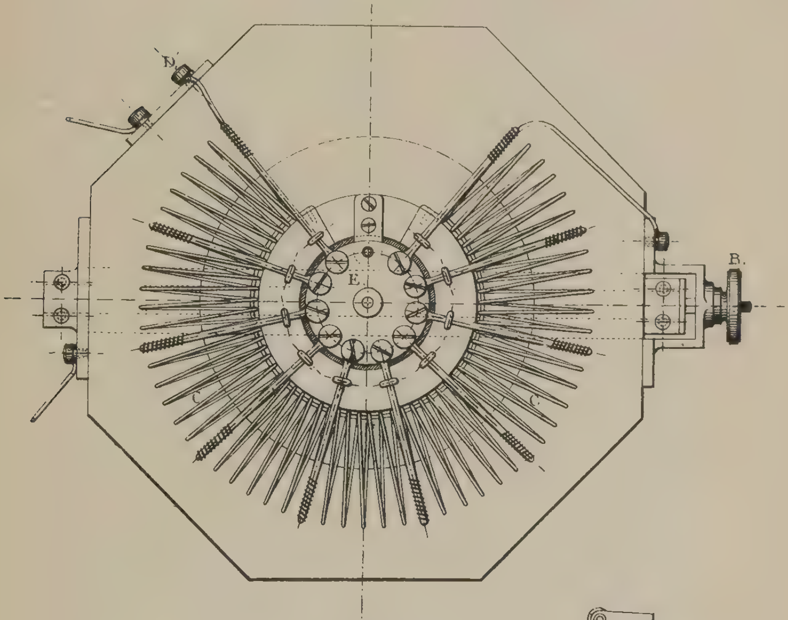
* Read February 22nd.

† The vertical fall being 150 feet, the increase of temperature would be $\frac{150}{772} = \frac{1}{5}^{\circ}$ Fahrenheit nearly.

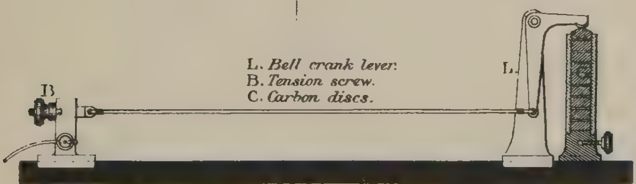
CURRENT REGULATOR.

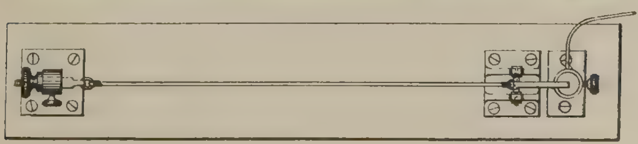
Scale 2/5 of full size.





L. *Bell crank lever.*
B. *Tension screw.*
C. *Carbon discs.*





cost of maintenance would be very small, and its power of transmitting electric energy would be limited only by the heat generated in it through electric resistance.

In venturing to give expression to my thoughts upon this subject, in my address to the Iron and Steel Institute in March 1877, I stated that a copper rod 3 inches in diameter would be capable of transmitting energy to the extent of a thousand horse-power an hour a distance of 30 miles, there to give motion to electrodynamic engines, or to produce illumination sufficient to light up a town with 250,000 candle-power.

Although this statement was considered by many a bold one at the time it was made, I now find that a conductor such as I then described might be able to transmit three or four times the amount of power then named, and that the light producible per horse-power was also, according to our present more advanced state of knowledge, very much understated.

No serious difficulty need be apprehended as to the production of a current sufficient in amount to fill a conductor of such large proportions as here indicated. Although it would perhaps be impossible to construct a single dynamo-electric machine of sufficient power for that purpose, any number of smaller machines could be easily coupled up both for intensity and quantity to produce the desired aggregate amount.

A difficulty would, however, arise at the other end, where the electric energy was to be applied, and where it would therefore be requisite to have an arrangement for its distribution over a number of branch circuits, so that each might receive such a proportion of the total current in the main conductor as to produce the number of lights, or the amount of power intended to be supplied. An accidental increase of resistance in one or other of the branch circuits would produce the double inconvenience of starving the circuits in which such increased resistance had occurred and of supplying an excess of current to the other circuits.

In order to carry out such a system of supply, it would be necessary to have the means of so regulating the current in each branch circuit, that only a predetermined amount should be allowed to flow through the same; it would be desirable also to furnish each circuit with the means of measuring and recording the amount of electric current passed through the same in any period of time.

It is my special purpose to bring before you an instrument by which these two purposes can be accomplished. The current-regulator (as represented in Plate XII.) consists principally of a strip of metal (of mild steel or fused iron by preference), which by its expansion and contraction regulates the current passing through it. This strip is rolled down to a thickness not exceeding 0·05 millim., and is of such a breadth that the current intended to be passed through the regulated branch circuit would raise the temperature of the strip to say 50° C.

This strip of metal (A) is stretched horizontally between a fixed support and a regulating-screw (B), at which latter the current enters, passing through the strip, and thence through a coil of German-silver wire (C) laid in the form of a collar round the centre, and connected at its other extremity with a binding-screw (D), whence the current flows on towards the lights or other apparatus to be worked by electricity. Upon its middle the strip carries a saddle of insulating material, such as ebonite, upon which rests a vertical spindle, supporting a circular metallic disk (E), with platinum contacts arranged on its upper surface. Ten or any other number of short stout wires connect the helical rheostat at equidistant points with adjustable contact-screws (F), standing above the platinum contacts on the surface of the metallic disk. These wires are supported upon the circular frame (G) of wood or other insulating material, but are free to be lifted off their support if the metallic disk should rise sufficiently to be brought into contact with the screws. These latter are so adjusted that none of them touches the metallic disk when it is in its lowest position, but that they are brought one after another into contact with the same as the disk rises; and it will be easily seen that for every additional contact-screw that is raised *seriatim* by the disk, a section of the helical rheostat between attachment and attachment is short-circuited by the metallic disk, and thus excluded from the circuit. When the disk is in its uppermost position the whole of the rheostat is short-circuited, and the regulator offers no other resistance to the current than that of the horizontal strip itself. In setting the regulator to work the regulating-screw (B) is drawn on sufficiently to bring the whole of the contact-screws into contact with the disk. The passage of the current through the

strip will have the effect of raising its temperature to an extent commensurate with the electrical resistance; and in the same measure the strip itself will be elongated, and cause the spindle with the contact-disk to descend.

Another form of this instrument depends for its action upon the circumstance discovered by the Count du Moncel in 1856, and more recently taken advantage of by Mr. Edison, that the electrical resistance of carbon varies inversely with the pressure to which it is subjected. A steel wire of 0·3 millim. diameter is attached at one end to an adjusting-screw, B, and at the other to one end of a bell-crank lever, L, by means of which the pressure is brought to bear upon a pile of carbon disks, C, placed in a vertical glass tube. The current enters the instrument at the adjusting-screw B, and, passing through the wire and bell-crank lever, leaves below the pile of carbon disks. Its effect is to cause a rise of temperature in the steel wire, which, through its expansion, diminishes the pressure upon the carbon disks, and thus produces an increase in their electrical resistance. This simple apparatus thus supplies a means of regulating the strength of small currents, so as to vary only within certain narrow limits.

According to Joule's law the heat generated in the strip per unit of time depends upon its resistance, and upon the square of the current; or

$$H=C^2R, \therefore C=\sqrt{\frac{H}{R}}.$$

On the other hand, the dissipation of heat by radiation depends upon the surface of the strip, and upon the difference between its temperature and that of the air. Therefore, in order that the current C may remain constant, it must, at every moment, be equal to the square root of the temperature divided by the resistance; and this function is performed automatically by the regulator, which throws in or takes out resistance in the manner described, according as the temperature increases or diminishes.

The regulating instrument may also be adapted to the measurement of powerful electric currents, by attaching to the end of the sensitive strip a lever, with a pencil pressing with its point upon a strip of paper drawn under it in a parallel direction with the lever by means of clockwork, a datum line being drawn on the strip by another pencil. The

length of the ordinate between the two lines depends, in the first place, upon the current which passes at each moment, and, in the second place, upon the loss of heat by radiation from the strip.

If R' is the resistance and H' the heat with a current C' and temperature T', then, by the law of Joule,

$$H' = R'C'^2,$$

and the loss by radiation is equal to

$$H' = (T' - T)S,$$

in which T' is the temperature of the strip, T that of the atmosphere, and S the surface of the strip.

Considering that the resistance varies as the absolute temperature of the conductor, according to a law first expressed by Helmholtz, the value of R may be put for R' for small variations of temperature; and as during an interval of constant current the heat generated and that radiated off will be equal, we obtain

$$C'^2 = (T' - T)\frac{S}{R}, \therefore C' = \sqrt{\frac{(T' - T)S}{R}} \quad . \quad . \quad . \quad (1)$$

in which $T' - T$ represents the movement of the pencil, and S is constant.

For any other temperature T'',

$$C'' = \sqrt{\frac{(T'' - T)S}{R}}.$$

For small differences of C'' and C',

$$(C'' - C')^2 = 2C''(C'' - C');$$

that is to say, small variations of current will be proportional to the variations in the temperature of the strip.

To determine the value of a diagram in Weber's or other units of current, it is only necessary, if the variations are not excessive, to average the ordinates, and to determine their value by equation (1), or from a Table.

These observations may suffice to show the possibility of regulating and measuring electric currents with an ease and certainty quite equal to that obtained in dealing with currents of liquids such as gas or water; and the time may not be far distant when the use of such an instrument will also become a public necessity.

Other forms of the instrument will readily suggest themselves to the mind of the constructive engineer; but the two typical forms I have described on this occasion will suffice, I think, to show its general character.